72. *Probability-theoretic Investigations on Inheritance. XI_2. Absolute Non-Paternity.*

By Yûsaku KOMATU.

Department of Mathemtatics, Tokyo Institute of Technology and
Department of Legal Medicine, Tokyo Medical and Dental University.

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4. Absolute non-paternity against brethren with different fathers.

Concerning brethren with different fathers, i.e., children with a mother alone in common, analogous problems arise as in the preceding section. We first consider a problem corresponding to the one discussed in § 2 of X. Let us denote by

$$D_0(ij,\ hk) \tag{4.1}$$

the probability of an event that a brethren combination (A_{ij}, A_{hk}) *with different fathers appears and then the proof of absolute non-paternity can be established against both of them.* This is the basic quantity corresponding to (2.2) of X. The explicit expression for (4.1) can immediately be derived from (2.1) by replacing merely a factor $\sigma(ij, hk)$ by the corresponding one $\sigma_0(ij, hk)$. We thus get, corresponding to (2.2) to (2.8), the following results:

$$D_0(ii,\ ii)=\tfrac{1}{2}p_i^3(1+p_i)(1-p_i)^2, \tag{4.2}$$

$$D_0(ii,hh)=\tfrac{1}{2}p_i^2p_h^2(1-p_i-p_h)^2 \qquad (h\neq i), \tag{4.3}$$

$$D_0(ii,\ ih)=\tfrac{1}{2}p_i^2p_h(1+2p_i)(1-p_i-p_h)^2 \qquad (h\neq i), \tag{4.4}$$

$$D_0(ii,hk)=p_i^2p_hp_k(1-p_i-p_h-p_k)^2 \qquad (h,k\neq i;\ h\neq k); \tag{4.5}$$

$$D_0(ij,\ ij)=\tfrac{1}{2}p_ip_j(p_i+p_j+4p_ip_j)(1-p_i-p_j)^2 \qquad (i\neq j), \tag{4.6}$$

$$D_0(ij,\ ih)=\tfrac{1}{2}p_ip_jp_h(1+4p_i)(1-p_i-p_j-p_h)^2 \qquad (i\neq j;\ h\neq i,j), \tag{4.7}$$

$$D_0(ij,hk)=2p_ip_jp_hp_k(1-p_i-p_j-p_h-p_k)^2 \qquad (i\neq j;\ h,k\neq i,j;\ h\neq k). \tag{4.8}$$

A symmetry relation corresponding to (2.9) is valid here also:

$$D_0(ij,hk)=D_0(hk,ij) \qquad (i,j,h,k=1,\ldots,m). \tag{4.9}$$

Partial sums corresponding to (2.10) and (2.11) become

$$\begin{aligned} D_0(ii)=p_i^2(&1-3S_2+\tfrac{5}{2}S_3+S_2^2-\tfrac{3}{2}S_4 \\ &-(2-3S_2+S_3)p_i+2(2-S_2)p_i^2-\tfrac{11}{2}p_i^3+\tfrac{7}{2}p_i^4), \end{aligned} \tag{4.10}$$

$$\begin{aligned} D_0(ij)=2p_ip_j(&1-3S_2+\tfrac{5}{2}S_3+S_2^2-\tfrac{3}{2}S_4 \\ &-(2-3S_2+S_3)(p_i+p_j)+2(2-S_2)(p_i^2+p_j^2)-2p_ip_j \\ &-\tfrac{11}{2}(p_i^3+p_j^3)-3p_ip_j(p_i+p_j)+\tfrac{7}{2}(p_i^4+p_j^4)+p_ip_j(p_i^2+p_j^2) \\ &+2p_i^2p_j^2) \qquad (i\neq j). \end{aligned} \tag{4.11}$$

Sub-probabilities over homo- and heterozygotic first children become