4. On the Structure of the Plane Translation of Brouwer

By Tatsuo HOMMA and Hidetaka TERASAKA

(Comm. by Z. SUETUNA, M.J.A., Jan. 12, 1953)

The so-called plane translation theorem of Brouwer[1] had been a starting point to a series of investigations concerning the sense preserving topological transformation of the Euclidean plane onto itself without fixed point[2][3][4][5][6][7]. But the general behavior of such a transformation, called by Scorza Dragoni[5] a generalized translation, is not so simple as one would expect at first sight and but little is definitely known until now. In this note we shall communicate without proof the result of our investigation which purports to determine the complete structure of the generalized translation, given in the main theorem at the end of this note[*].

1. Throughout this note f denotes a sense preserving topological transformation of the Euclidean plane E^2 onto itself without fixed point. The n-th iteration of f will be denoted by f^n for any integer n.

2. If M is a point set, the cluster set of $f^n(M)$ for all positive integer n will be called the *$(+)$-limit* of M and denoted by $\lim^+ M$; likewise for *$(-)$-limit*.

If $\lim^+ M$ is non vacuous, then M is said to be *$(+)$-irregular*. A point p is said to be *$(+)$-irregular*, if every neighbourhood $U(p)$ of p, where we understand by a neighbourhood always a domain containing the point in question, is $(+)$-irregular. If furthermore $P=\bigcup \lim^+ U(p)$ for all neighbourhoods $U(p)$ of p is non vacuous, then p is said to be *strongly $(+)$-irregular*. If P vanishes, then p is said to be *weakly $(+)$-irregular*.

Similarly for *$(-)$-irregularity*[**].

If a point p is neither $(+)$-nor $(-)$-irregular, then p is said to be *regular*. We can construct a simple *example of f for which all points of the plane are both $(+)$- and $(-)$-irregular*.

$P=\bigcup \lim^+ U(p)$ will be called the *$(+)$-singularity polar to p*, and p a *pole* of P. Similarly for *$(-)$-singularity*[***].

Duality Theorem. If p is strongly $(+)$-irregular, then every point q of the $(+)$-singularity P polar to p are strongly $(-)$-irregu-

---

*) Full account will appear in Osaka Math. Jour., **5**.

**) Since a proposition remains true if we interchange $(+)$ and $(-)$, we omit in the sequel the propositions thus obtained.

***) The notion of singularity has already been mentioned by Sperner[6].