2. On Homotopy Classification and Extension

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In the present note we give a homotopy classification theorem for mappings of an $(n+k)$-dimensional complex into a finite complex Y such that

(1) $\qquad \pi_i(Y)=0$ for $0\leqq i<n$ and $n<i<n+k$,

and the corresponding extension theorem, where $n\geqq k+2$ and $k\leqq 6$.[1)2)]

§1. Let $X(\ni x_0)$ be an arcwise connected and simply connected space, and let $X^*=X\cup e^{n_1}\cup\cdots\cup e^{n_s}$, where the boundary $\dot{e}^{n_i}(\ni q_i)$ of each cell e^{n_i} is attached to X by a map $f_i:(\dot{e}^{n_i},q_i)\to(X,x_0)$. We refer to such a space X^* as $\{X|e^{n_1},\cdots,e^{n_s};f_1,\cdots,f_s\}$. Suppose that $g_i:(E^r,\dot{E}^r,p_0)\to(e^{n_i},\dot{e}^{n_i},q_i)$ $(i=1,\cdots,t\leqq s)$ is a representative of the homotopy class $\{g_i\}\in\pi_r(e^{n_i},\dot{e}^{n_i},q_i)$, and that a condition $\sum_{i=1}^{t}\{f_i\circ(g_i|\dot{E}^r)\}=0$ is satisfied in $\pi_{r-1}(X,x_0)$. Then we can construct a map of an r-sphere S^r in X^* as follows: Let ε_i^r $(i=1,2,\cdots,t)$ be t disjoint r-cells in S^r which have a single point p in common, and let $\varepsilon^r=\bigcup_{i=1}^{t}\varepsilon_i^r$, $\dot{\varepsilon}^r=\bigcup_{i=1}^{t}\dot{\varepsilon}_i^r$. Choose an orientation of S^r, and orient each ε_i^r in agreement with S^r. If we map each ε_i^r to e^{n_i} by the map g_i, we get a map $g':(\varepsilon^r,\dot{\varepsilon}^r,p)\to(X^*,X,x_0)$ such that $g'|\dot{\varepsilon}^r$ is null-homotopic in X. Map now $\overline{S^r-\varepsilon^r}$ in X by an arbitrary null-homotopy of $g'|\dot{\varepsilon}^r$, then we obtain a map of S^r in X^*. This is the desired map and such a map is denoted by $<g_1,g_2,\cdots,g_t|X>$.

As for the spherical-maps, we use the following notations: $i_r:S^r\to S^r$ $(r\geqq 1)$ is the identity map; $\eta_r:S^{r+1}\to S^r$ $(r\geqq 2)$, $\nu_r:S^{r+3}\to S^r$ $(r\geqq 4)$ are the suspensions of the Hopf maps η_2, ν_4 respectively. Let $\partial_n:\pi_{n+1}(e^{r+1},\dot{e}^{r+1})\approx\pi_n(S^r)$ be the homotopy boundary, then we refer to maps in the homotopy classes $\partial_r^{-1}\{i_r\}$, $\partial_{r+1}^{-1}\{\eta_r\}$, $\partial_{r+3}^{-1}\{\nu_r\}$ as $\bar{i}_{r+1}$, $\bar{\eta}_{r+1}$, $\bar{\nu}_{r+1}$ respectively.

§2. Using the homology theory of Abelian groups due to Eilenberg—MacLane[3)] and the known results relative to the homotopy

1) Full details will appear in the Journal of the Institute of Polytechnics, Osaka City University. The first part of the details was already presented to the editor of the journal.

2) A general theory of this problem was given by S. Eilenberg—S. MacLane (cf. Proc. Nat. Acad. Sci., U.S.A., IV).

3) S. Eilenberg—S. MacLane: Cohomology theory of Abelian groups and homotopy theory II. Proc. Nat. Acad. Sci., U.S.A., **36**, No. 11 (1950); IV ibid., **38**, No. 4 (1952).