172. On the Uniqueness of the Cauchy Problem for Semi-elliptic Partial Differential Equations. II

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5. Proof of Theorem 1. At first we note that once (2.1), (4.1), and (4) in Lemma 2 are established Theorem 1 can be proved by arguments parallel with them in [1]. Let $V(N^0)$ and $U(0)$ be neighborhoods of N^0 and $x=0$ respectively in which both (2.1) and (4.1) are verified. For any $N_g \in V(N^0)$ and $x_g \in U(0)$ fixed, and $u_g \in C_0^\infty(U(0))$, multiplying $\hat{u}_g(\xi+i\tau N_g)$, which is a translation of Fourier transform of u_g, the both sides of (2.1) and (4.1), and applying Parseval formula, we obtain

$$(5.1)\qquad \sum_{j=1}^{n}\sum_{(1-\frac{1}{m_j})}\int |D^\alpha u_g|^2 \exp(2\tau\langle x, N_g\rangle)\,dx \le C\int\left\{\sum_{j=1}^{n}|P_0^{(j)}(x_g, D)u_g|^2+|u_g|^2\exp(2\tau\langle x, N_g\rangle)\right\}dx$$

$$(5.2)\qquad \sum_{(1)}\int |D^\alpha u_g|^2\exp(2\tau\langle x, N_g\rangle)\,dx \le D\int\Big\{|P_0(x_g, D)u_g|^2 + |\tau|^2|N_g|^2|P_0^{(1)}(x_g, D)u_g|^2\exp(2\tau\langle x, N_g\rangle)\Big\}dx,^{*)}$$

where $\langle x, N_g\rangle$ is $\sum_{j=1}^{n} x_j N_{gj}$.

To replace the weight function $\exp(\langle x, N_g\rangle)$ by $\exp(\varphi_\delta(x))$ in the above, we use a partition of unity designed by Hörmander so that in each corresponding neighborhood $\varphi_\delta(x)$ is almost equal to a linear function. That is:

$$\omega(x)\in C_0^\infty(x; \forall i, |x_i|<1),\ \omega(x)\neq 0 \text{ on } \left(x; \forall i, |x_i|\leqq \frac{1}{2}\right)$$

$g=(g_1, g_2, \cdots, g_n)$; g_i's vary in all integers,

$$\theta(x)=\frac{\omega(x)}{\sum_g \omega(x-g)},\quad \theta_g(x)=\theta(x_1-g_1, x_2-g_2, \cdots, x_n-g_n),$$

and for $u\in C_0^\infty(\Omega)$, $u(x)=\sum_g \theta_g(x)u(x)$.

On a support of $\theta_g(x)$, $\varphi_\delta(x)\le\varphi_\delta(x_g)+\langle x-x_g, N_g\rangle\le\varphi_\delta(x)+n\tau^{-1}$ holds where N_g equals to grad $\varphi_\delta(x_g)$. Then for $\tau>\frac{1}{2}$, and $C_1=\exp(2n)C$, $D_1=\exp(2n)D$, we get

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*) So far as we can avoid confusion, we use the same letters D, C, etc. for other constants.