171. On the Uniqueness of the Cauchy Problem for Semi-elliptic Partial Differential Equations. I

By Akira TSUTSUMI

University of Osaka Prefecture

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1. Introduction. In this note we shall prove the inequalities of Carleman type from which we can derive the uniqueness of the Cauchy problem with data on a noncharacteristic surface, having a restriction on its curvature, for some class of semi-elliptic equations. For parabolic equations which are typical in semi-elliptic equations; $\left(\frac{\partial}{\partial t}-L\right)u=0$ (L: 2nd order elliptic operator) M. H. Protter proved the uniqueness when data are given on a time-like surface, (see [5]), S. Mizohata proved it when data are given on any hyperplane not orthogonal to t-axis, (see [4]), and H. Kumanogo generalized the result of Mizohata (see [3]). For elliptic equations which are also typical in semi-elliptic, L. Hörmander proved the uniqueness under mild assumptions. (See [1].)

On the other hand L. Hörmander showed that for any integer $r\geqq 1$ there are examples of non-uniqueness; $\left\{\left(\frac{1}{i}\frac{\partial}{\partial x_2}\right)^r+a(x_1, x_2)\frac{\partial}{\partial x_1}\right\}u=0$, $a(x_1, x_2)=0$ for $x_2\leq 0$. These have several means, but at a point of view of the type of equations these are not semi-elliptic at the origin. (See [2].) This is our motive to study the uniqueness for semi-elliptic equations of higher order. Main tools of our proof are the partition of unity of Hörmander and the inequality of Trèves which is extended for our operators. (See [1], [6].)

2. Notations and some class of semi-elliptic operators. $x=(x_1, x_2, \cdots, x_n)$ is a variable point of n-dimensional euclidean space R^n, and $\xi=(\xi_1, \xi_2, \cdots, \xi_n)$ is a vector of Ξ^n dual to R^n, and $\tilde{\xi}$ denotes a vector $(\xi_2, \xi_3, \cdots, \xi_n)$. m is a vector $(m_1, m_2, \cdots, m_n)$ where m_j's are positive integers, α is a vector $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ where α_j's are non-negative integers, by $|\alpha:m|$ we denote $\sum_{j=1}^n \alpha_j/m_j$, $|\alpha|$ is a length of α; $\sum_{j=1}^n \alpha_j$, and m_0 is the minimum of m_j. ξ^α is $\xi_1^{\alpha_1}, \xi_2^{\alpha_2}\cdots\xi_n^{\alpha_n}$. A polynomial of ξ whose coefficients are functions of x can be written in the following form.

$$P(x,\xi)=P_0(x,\xi)+Q(x,\xi),$$
$$P_0(x,\xi)=\sum_{|\alpha:m|=1} a_\alpha(x)\xi^\alpha,\quad Q(x,\xi)=\sum_{j=1}^n \sum_{|\alpha:m|\leq 1-\frac{1}{m_j}} a_\alpha(x)\xi^\alpha.$$