# 141. On Some Results Involving Jacobi Polynomials and the Generalized Function $\tilde{\omega}_{\mu_1,\cdots,\mu_n}(x)$

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**Abstract.** The object of this paper is to evaluate the following type of multiple integrals:

$$\prod_{r=1}^{m}\int_0^1 x_r^{\rho_r}(1-x_r)^{\beta_r}P_{n_r}^{(\alpha_r,\beta_r)}(1-2x_r)dx_r\tilde{\omega}_{\mu_1,\cdots,\mu_n}[\lambda(x_1\cdots x_m)^{\pm h/2}].$$

These integrals are then employed to establish the expansions for the $\tilde{\omega}_{\mu_1,\cdots,\mu_n}(x)$ function involving Jacobi polynomials.

**1. Introduction.** The function $\tilde{\omega}_{\mu_1,\cdots,\mu_n}(x)$ was defined [1] by the integral equation

$$(1.1)\qquad \tilde{\omega}_{\mu_1,\cdots,\mu_n}(x)=x^{1/2}\int_0^\infty\cdots\int_0^\infty J_{\mu_1}(t_1)\cdots J_{\mu_{n-1}}(t_{n-1})J_{\mu_n}\left(\frac{x}{t_1\cdots t_{n-1}}\right)\cdot(t_1\cdots t_{n-1})^{-1}dt_1\cdots dt_{n-1},$$

$$=\int_0^\infty\tilde{\omega}_{\mu_1,\cdots,\mu_{n-1}}(x/t)J_{\mu_n}(t)t^{-1/2}dt$$

Where $R\left(\mu_k+\frac{1}{2}\right)\geq 0, k=1,2,\cdots,n$ and $\mu$'s may be permuted among themselves.

The following results are known.

(1.2) $\tilde{\omega}_\mu(x)=\sqrt{x}J_\mu(x),\quad \tilde{\omega}_{\mu,\mu+1}(x)=J_{2\mu+1}(2\sqrt{x}),\quad R(\mu)>-1.$

(1.3) The Mellin transform of $\tilde{\omega}_{\mu_1,\cdots,\mu_n}(x)$ is

$$2^{n(s-1/2)}\cdot\frac{\Gamma\left(\frac{\mu_1}{2}+\frac{s}{2}+\frac{1}{4}\right)\cdots\Gamma\left(\frac{\mu_n}{2}+\frac{s}{2}+\frac{1}{4}\right)}{\Gamma\left(\frac{\mu_1}{2}-\frac{s}{2}+\frac{3}{4}\right)\cdots\Gamma\left(\frac{\mu_n}{2}-\frac{s}{2}+\frac{3}{4}\right)}.$$

In this paper we have evaluated some multiple integrals involving the above generalized function and empolyed them to obtain some expansion formulae for the generalized function $\tilde{\omega}_{\mu_1,\cdots,\mu_n}(x)$. Particular cases have also been given with proper choice of parameters.

**2. The multiple integrals.** The integrals to be evaluated are:

$$(2.1)\qquad \prod_{r=1}^{m}\int_0^1 x_r^{\rho_r}(1-x_r)^{\beta_r}P_{n_r}^{(\alpha_r,\beta_r)}(1-2x_r)dx_r\tilde{\omega}_{\mu_1,\cdots,\mu_n}[\lambda(x_1\cdots x_m)^{\pm h/2}]$$

$$=\frac{h^{-\Sigma\beta_r-1}}{\pi 2^{n/2}}\prod_{r=1}^{m}\left(\frac{\Gamma(\beta_r+n_r+1)}{\Gamma(n_r+1)}\right)\sum_{i,-i}\frac{1}{i}G^{mh+1,mh+n+1}_{2n+2mh+1,2mh+1}$$

$$\times\left(\frac{2^{2n}e^{i\pi}}{\lambda^2}\middle|\begin{matrix}\left(\frac{3}{4}-\frac{\mu_j}{2}\right)_n, \Delta(h,\rho_j-\alpha_j-n_j+1)_m, 1,\\ \Delta(h,\rho_j+1)_m, 1,\end{matrix}\right.$$