206. *Note on Commutative Regular Ring Extensions of Rings*

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Let R be a commutative ring with identity, and let S be a commutative ring extension of R with the same identity. If the canonical injection $R \rightarrow S$ is a flat epimorphism (in the category of commutative rings with identity), then, by Lazard [3], for any ideal B of S

$$B \rightarrow B \cap R \tag{$*$}$$

is an injective mapping from the set of ideals of S into that of R.

In case S is a regular (in the sence of von Neumann) ring extension of R, we can give certain conditions that are necessary and sufficient for the mapping $(*)$ to be injective.

It is easily seen that if S is the classical ring of quotients of R the mapping $(*)$ is injective. There is an example of a commutative ring extension of R for which the mapping $(*)$ is a bijection but which does not coincide with the classical ring of quotients of R.

Throughout this note, a ring considered will mean a commutative ring with identity and its ring extension will mean a commutative one with the same identity.

1. Let R be a ring. For its ring extension S, we shall consider the following conditions:

(α) $B=(B \cap R)S$ *for any ideal B of S.*

(β) $A=AS \cap R$ *for any ideal A of R.*

The conditions (α) and (β) that the mapping $(*)$ are injective and surjective, respectively.

For a regular ring extension S of R, the spectrum X of S, i.e. the space of prime (=maximal) ideals with the hull-kernel topology, is compact, Hausdorff and extremely disconnected and S may be identified with the ring of global sections of a sheaf of fields over X (see [6] for a detailed discussion).

For $s \in S$ and $x \in X$, let s_x be the image of s under the natural homomorphism of S onto S/x, and let $S(s)$ be the support of s, i.e.

$$S(s)=\{x \in X \mid s_x \neq 0\}=\{x \in X \mid s \notin x\}.$$

Now we shall quote Mewborn [5, Theorem 3.1] as follows.

Lemma 1.1. *Let R be a ring and let S be a regular ring extension of R. Then S is flat as an R-module if and only if for any $a \in R$, there exists a finitely generated ideal I in R such that*