204. On Potent Rings. II

By Hidetoshi MARUBAYASHI
College of General Education, Osaka University

(Comm. by Kenjiro SHODA, M. J. A., Oct. 12, 1970)

In [7], we defined residue-finite *CPI*-rings which are s-complemented with respect to L_{r2}^*. In this paper we shall give characterizations of such rings. Let R be a residue-finite *CPI*-ring and let $\hat{R}$ be the maximal right quotient ring of R. We shall give also a necessary and sufficient condition that $\hat{R}$ is a left quotient ring of R. This is a generalization of Faith's result [1] on prime rings. Terminology and notation will be taken from [6] and [7].

1. Triangular-block matrix rings with infinite dimension.

We shall give examples of residue-finite *CPI*-rings which are s-complemented with respect to L_{r2}^*. Let F be a division ring and let ω be a countable ordinal number. We denote by $(F)_\omega$ the ring of all column-finite $\omega\times\omega$ matrices over F. Let F_{ij} be additive subgroups of F such that

(1.1) $\qquad F_{ij}F_{jk}\subseteqq F_{ik} \qquad (i,j,k=1,2,\cdots).$

Let

(1.2) $\qquad S=\{a\in(F)_\omega \mid a=(a_{ij}),\, a_{ij}\in F_{ij}\}.$

Clearly S is the subring of $(F)_\omega$. The ring S will be called a T-ring (triangular-block matrix ring) with type (A) in $(F)_\omega$ iff there exist integers $0=d_0<d_1<\cdots<d_n<\cdots$ such that

(1.3) $F_{ij}\neq 0$ iff $i>d_p$ and $d_p<j\leqq d_{p+1}$ $(p=0,1,2,\cdots)$.

The ring S will be called a T-ring with type (B) in $(F)_\omega$ iff there exist integers $0=d_0<d_1<\cdots<d_p$ such that

(1.4) $F_{ij}\neq 0\Longleftrightarrow$(i) if $j\leqq d_p$, then $i>d_k$ and $d_{k-1}<j\leqq d_k$ for some $k(1\leqq k\leqq p)$, (ii) if $j>d_p$, then $i>d_p$.

In both cases, associated with S is the full T-ring

(1.5) $M=\{a\in(F)_\omega \mid a=(a_{ij}),\, a_{ij}\in F'_{ij}\}$, where $F'_{ij}=F$ whenever $F_{ij}\neq 0$ and $F'_{ij}=0$ otherwise.

Following R. E. Johnson, we shall call M the full cover of S. Let A and B be subsets of a division ring F. The set $\{ab^{-1}\mid a\in A,\, 0\neq b\in B\}$ will be denoted by AB^{-1}. A ring Q is called a right quotient ring of a subring R if for each $a, 0\neq b\in Q$, there exist $r\in R$ and $n\in Z$ such that $ar+na\in R$ and $br+nb\neq 0$, where Z is the ring of integers; in symbols: $R\leqq Q$. A left quotient ring is defined similarly. If Q is a left and right quotient ring of a ring R, then we write $R\leqq_l Q$. If R has the zero right singular ideal, then Q is a right quotient ring of R if and only if Q is