# 252. *On Wiener Functions of Order m*

By Hidematu TANAKA
Mathematical Institute, Nagoya University

(Comm. by Kinjirô KUNUGI, M. J. A., Dec. 12, 1970)

**1.** Let $\Omega$ be an open subset of the $n$-dimensional Euclidian space $R^n (n \geq 2)$ and $f_i$ be a continuous function on the boundary of $\Omega (1 \leq i \leq m)$. Riquier's problem for a polyharmonic equation $\Delta^m u = 0$ on $\Omega$ is to find a function $u$ such that $\Delta^m u = 0$ in $\Omega$ and $(-\Delta)^{i-1} u = f_i$ on the boundary of $\Omega$ for each $i (1 \leq i \leq m)$.

For a unit disk it was solved by Riquier and for a bounded open set by M. Itô [2].

In this note we shall show that for an unbounded open subset $\Omega$ its problem can be solved by means of Wiener ideal boundary $\Delta_W$ and Wiener harmonic boundary $\Gamma_W$ of $\Omega$ (Theorem 3).

Let $f_i$ be a continuous function on $\Delta_W (1 \leq i \leq m)$. Then there exists a function $h_{(f_1, f_2, \cdots, f_m)}$ on $\Omega$ such that

$$\Delta^m h_{(f_1, f_2, \cdots, f_m)} = 0$$

in $\Omega$ and for each $i$ $(1 \leq i \leq m)$, on $\Gamma_W$

$$(-\Delta)^{i-1} h_{(f_1, f_2, \cdots, f_m)} = f_i$$

if and only if $\Omega$ satisfies the condition

$$\int G_\Omega^{(m-1)}(x, y) dy < +\infty$$

for some point $x$ in $\Omega$, where $G_\Omega$ being the Green function of $\Omega$,

$$G_\Omega^{(m-1)}(x, y) = \int \cdots \int G_\Omega(x, z_1) G_\Omega(z_1, z_2) \cdots G_\Omega(z_{m-2}, y) dz_1 dz_2 \cdots dz_{m-2}.$$

**2.** Let $\Omega$ be an open subset of $R^n$. We call a real valued function $u$ in the class $C^{2m}(\Omega)$ is polyharmonic of order $m$ in $\Omega$ if we have in $\Omega$

$$\Delta^m u = \left( \sum_{k=1}^{n} \frac{\partial^2}{\partial x_k^2} \right)^m u = 0.$$

For the Green function $G_\Omega$ of $\Omega$ and an integer $i \geq 1$, we put

$$G_\Omega^{(i)}(x, y) = \int \cdots \int G_\Omega(x, z_1) G_\Omega(z_1, z_2) \cdots G_\Omega(z_{i-1}, y) dz_1 dz_2 \cdots dz_{i-1}.$$

By a suitable normalization we have $(-\Delta_y)^i G_\Omega^{(i)}(x, y) = \varepsilon_x$ in $\Omega$, where $\varepsilon_x$ is the Dirac measure at $x$.

From now on, let $m (\geq 1)$ be a fixed integer and $i$ be any integer $1 \leq i \leq m$. As to the solution of Riquier's problem, M. Itô [2] proved

**Lemma 1.** *Let $\Omega$ be a bounded open subset of $R^n$ and $(f_i)_{i=1}^m$ be a system of bounded continuous functions on the boundary $\partial \Omega$ of $\Omega$. Then there exist $m$ positive Radon measures $\varepsilon_{x, c\Omega}^{(i)} (1 \leq i \leq m)$ on $\partial \Omega$, and the function*