251. *Some Existence Theorems in Cluster Set Theory*

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1. Let C be the unit circle and D be the open unit disk in the complex plane.

Theorem 1. *There exists a holomorphic function f in D for which the set $I(f)$ of Plessner points* [3, p. 147] *is residual* [3, p. 75] *on C and of logarithmic measure* [7, p. 64] *zero.*

Theorem 2. *There exists a bounded univalent holomorphic function f in D for which the set $M(f)$ of Meier points* [3, p. 153] *is of logarithmic measure zero.*

Furthermore, we obtain some improvements of Bagemihl-Seidel's results [2, p. 191, Corollaries $3\sim5$], one of which may be stated as

Theorem 3. *There exist a holomorphic function f in D and a subset S of C, being of logarithmic measure zero, such that the radial cluster set* [3, p. 72] *of f at any point of $C-S$ coincides with the unit circle.*

Remark 1. A bounded set of logarithmic measure zero is known to be of logarithmic capacity zero. In Remark 3 of the next section we ascertain this for our special example S.

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2. We construct a subset S of C satisfying the following three conditions:

(i) $C-S$ is of first Baire category on C.

(ii) S is a G_δ subset of C.

(iii) The logarithmic measure of S is zero.

Let $K=\{z_1,\cdots,z_n,\cdots\}$ be a countable dense subset of C and let $\varepsilon_1,\cdots,\varepsilon_k,\cdots$ be a sequence of positive numbers such that $\varepsilon_k\to0$ as $k\to\infty$. Let δ_{kn} be an open disk containing z_n whose radius is $r_{kn}=\exp(-2^n/\varepsilon_k)$ $(k,n=1,2,\cdots)$. Let $\delta_k=\bigcup_{n=1}^\infty\delta_{kn}$ and let $\delta=\bigcap_{k=1}^\infty\delta_k$. Then $S=\delta\cap C$ is the desired one. Indeed, for any k, the closed set $C-\delta_k$ is nowhere dense on C since $\delta_k\cap C\supset K$ is open and dense on C. Therefore the set

$$(*)\qquad C-S=\bigcup_{k=1}^\infty(C-\delta_k)$$

is of first category on C. To prove (iii) we use the same notation as in [7, p. 63 ff.] with $h(t)=\{\log(1/t)\}^{-1}$. We use "disks" instead of