244. *A Criterion for Boundedness of a Linear Map from any Banach Space into a Banach Function Space**)

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(Comm. by Kinjirô KUNUGI, M. J. A., Dec. 12, 1970)

[**Abstract.** Let (X, Λ, μ) be a totally sigma-finite complete measure space, ρ be a function norm with associate seminorm ρ', Y be a Banach space. If $L\rho$ is a Banach function space then for a linear mapping $T: Y \to L\rho$ be continuous it is necessary and sufficient that given $E \in \Lambda$ with $\rho'(\chi_E) < \infty$ the functional T_E defined by $T_E y = \int (Ty)(x)\chi_E(x)d\mu(x)$ is continuous. It is noted that the collection $\{E \in \Lambda : \mu(E), \rho'(\chi_D) < \infty\}$ is sufficient to generate the same integration theory as Λ and if ρ satisfies the Fatou property this collection even generates (algebraically and isometrically) the function space $L\rho$.]

This note is based entirely on the notes of Luxemburg and Zaanen [13], a knowledge of which, will be assumed throughout; the notations of those authors will be preserved and references to [13] Will simply note the particular results of [13] without further modification. Of course, reference to papers other than [13] will be modified by the appropriate reference list number.

Theorem. *Let ρ be a function norm satisfying the Riesz-Fischer property (so $L\rho$ is a Banach function space); suppose that Y is a Banach space and that $T: Y \to L\rho$ is a linear mapping.*

Then in order that T be continuous it is necessary and sufficient that the following hold: given $E \in \Lambda$ such that $\chi_E \in L\rho'$, the linear functional T_E defined on Y to the scalar field by

$$T_E y = \int_E (Ty)(x)d\mu(x)$$

be a mumber of Y'.

Proof. Necessity follows immediately from Lemma 13.1.

To prove sufficiency, we note that since ρ is a function norm it follows from Corollary 11.5 that ρ' is saturated (in fact, ρ's being a norm is equivalent to ρ''s being saturated), so that by Theorem 8.7 there exists a sequence of subsets X_n of X satisfying $X_n \nearrow X$, $\mu(X_n) < \infty$, and $\rho'(\chi_{X_n}) < \infty$ (of course, $X_n \in \Lambda$; for the rest of the proof we will assume the sequence $\{X_n\}$ to be chosen according to these requirements.

We now consider the linear mapping $T: Y \to L\rho$. We will show

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*) The research for this paper was supported in part by West Georgia College Faculty Research Grant No. 699.