195. The Theory of Nuclear Spaces Treated by the Method of Ranked Space. III

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(Comm. by Kinjirô KUNUGI, M. J. A., June 12, 1971)

In the papers [3] and [4], we defined the neighbourhood having a rank in the nuclear space Φ, and then we made the Φ a linear ranked space.

§ 4. The completion of the linear ranked space Φ, (1).

Definition 4. We say that a sequence $\{g_n\}$ in Φ is R-convergence having a limiting point zero, if there exists a fundamental sequence of neighbourhoods $\{V_{r_i}(0, r_i, m_i)\}$ such that $g_i \in V_{r_i}(0, r_i, m_i)$ for all i. And we denote it by $g_n \xrightarrow{R} 0$.

In the paper [4], we defined the equivalence of two R-Cauchy sequences in Φ, so that the set of all R-Cauchy sequences in Φ is divided into equivalence classes. We denote by $\hat{\Phi}$ the set of all these equivalence classes.

Now, suppose $\hat{g}, \hat{f} \in \hat{\Phi}$ and let $\{g_n\}$ and $\{f_n\}$ be two R-Cauchy sequences in Φ which are in the equivalence classes $\hat{g}$ and $\hat{f}$, respectively. Then $\{g_n+f_n\}$ is an R-Cauchy sequence. Moreover if $\{g'_n\}$ and $\{f'_n\}$ are R-Cauchy sequences equivalent to $\{g_n\}$ and $\{f_n\}$ respectively, then $\{g'_n+f'_n\}$ is equivalent to $\{g_n+f_n\}$. Thus we can define $\hat{g}+\hat{f}$ as the equivalence class which contains $\{g_n+f_n\}$, and the definition depends only on $\hat{g}, \hat{f}$, not on the particular choice of $\{g_n\}$, $\{f_n\}$. Likewise, for any scalar λ, we define $\lambda\hat{g}$ as the equivalence class which contains $\{\lambda g_n\}$. The zero element of $\hat{\Phi}$ is the unique equivalence class all of whose members $\{g_n\}$ are such that $g_n \xrightarrow{R} 0$.

Now, we shall define a neighbourhood with rank i in $\hat{\Phi}$.

Definition 5. We define a neighbourhood, $\hat{V}_i(0, r, m)$, of the origin in $\hat{\Phi}$. $\hat{g} \in \hat{V}_i(0, r, m)$ means that for an R-Cauchy sequence $\{g_n\}$ belonging to $\hat{g}$, there exist some number r', $0<r'<r$ and some integer N such that the relation $n \geqq N$ implies $g_n \in V_i(0, r', m)$. And we call $\hat{V}_i(0, r, m)$ a neighbourhood with rank i of the origin in $\hat{\Phi}$.

Moreover we define that the neighbourhood with rank 0, which is denoted by $\hat{V}_0$, is always the space $\hat{\Phi}$.

We shall show that in the definition above, every R-Cauchy sequence $\{g_n\}$ belonging to $\hat{g}$ has some number r', $0<r'<r$ and some integer N such that the relation $n \geqq N$ implies $g_n \in V_i(0, r', m)$.