181. A Criterion for the Existence of the Twosided Unity Element of Semigroups*)

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In this paper, which is a modified version of author's earlier article [4], written in Hungarian, will be given a necessary and sufficient condition, and its three corollaries, for the existence of the twosided unity element of a semigroup. Author's this criterion is left-right symmetric, and it can be transformed, almost trivially, also into a left-right nonsymmetric formally milder condition, such that it remains yet equivalent to the existence of the twosided unity element of the semigroup.

On the other hand, this criterion is an analogy of one among author's [5] criteria for the existence of the twosided unity element of an associative ring. Further criteria, for the existence of the unity element of a semigroup, were earlier discussed, in the joint paper [2] of S. Lajos and J. Szép, using the notion of so called magnifying elements.

The here used fundamental notions can be found e.g. in A. H. Clifford's and G. P. Preston's book [1] or in Ljapin's book [3]. Let S be an arbitrary semigroup, generally without zero element. An element r of a semigroup S with zero element z will be called right annihilator of S, if $sr=z$ for any $s\in S$ holds. Furthermore, the element r_1 of S is said to be right regular in S, if $xr_1=yr_1$ always implies $x=y$ $(x, y\in S)$. Left annihilator and left regularity are defined left-right dually. The center C of S is the set of all elements c, satisfying $cs=sc$ for any element $s(\in S)$, of S. Now let T be S itself, if S has zero z, and $T=S\cup z$ with $z^2=zs=sz=z$ for any $s\in S$, if $z\notin S$. T is called here the related semigroup of S. Then we have:

Theorem. *For an arbitrary semigroup S the following two conditions are equivalent*:

(I) *S has twosided unity element*;

(II) *S contains a right regular element r and a left regular element l, satisfying $rS\subseteq Sr$ and $Sl\subseteq lS$, furthermore the related semigroup T yet satisfies both of requirements*:

(*) *T has no homomorphic image with nonzero right annihilators*;

*) To Professor Jenö Szép on his 50th birthday.