230. *On the Radon Transform of the Rapidly Decreasing Functions on Symmetric Spaces*

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1. Let S be a Riemannian globally symmetric space and $\hat{S}$ the Radon dual space of S consisting of the holocycles in S. The purpose of this paper is to study the relations between the Schwartz functions on S and those on $\hat{S}$, that is, to study an $\mathcal{S}$-theory in a sense. For the detailed proof, see [1].

2. The Schwartz spaces. Let G denote the largest connected group of isometries of S in compact open topology. Let o be any point in S, K the isotropy subgroup of G at o and $\mathfrak{k}_0$ and $\mathfrak{g}_0$ their Lie algebras, respectively. Let $\mathfrak{g}_0=\mathfrak{k}_0+\mathfrak{p}_0$ be the corresponding Cartan decomposition of $\mathfrak{g}_0$. Let $\mathfrak{h}_{\mathfrak{p}_0}$ denote a Cartan subalgebra for the space S, $A_{\mathfrak{p}}$ the analytic subgroup of G corresponding to $\mathfrak{h}_{\mathfrak{p}_0}$ and M the centralizer of $\mathfrak{h}_{\mathfrak{p}_0}$ in K. Let extend $\mathfrak{h}_{\mathfrak{p}_0}$ to a Cartan subalgebra $\mathfrak{h}_0$ of $\mathfrak{g}_0$, of the corresponding roots let P_+ denote the set of those whose restriction to $\mathfrak{h}_{\mathfrak{p}_0}$ is positive in the ordering defined by a fixed Weyl chamber C in $\mathfrak{h}_{\mathfrak{p}_0}$. Then we obtain an Iwasawa decomposition $G=KA_{\mathfrak{p}}N$. Put $\rho=\frac{1}{2}\sum_{\alpha\in P_+}\alpha$ as usual. Let $\boldsymbol{D}(S)$(resp. $\boldsymbol{D}(\hat{S})$) denote the algebra of G-invariant differential operators on S (resp. $\hat{S}$) and $\hat{\boldsymbol{D}}$ the image of the isomorphism of $\boldsymbol{D}(S)$ into $\boldsymbol{D}(\hat{S})$.

For $x\in S=G/K$ and $g\in G$ such that $\pi(g)=x$ by the natural mapping π of G onto G/K, there exists a unique element $X\in\mathfrak{p}_0$ such that $x=\exp X\cdot K$. Now put

$$\omega(x)=\{\det(\sinh \operatorname{ad} X/\operatorname{ad} X)\}^{1/2},$$
$$\sigma(g)=\sigma(x)=\|X\|,$$
$$\xi(x)=\int_K \exp\{-\rho(H(\exp X\cdot k))\}dk.$$

For $f\in C^\infty(S)$, $D\in\boldsymbol{D}(S)$ and integer $d\geqq 0$, put

$$\nu_{D,d}(f)=\sup_S |Df|(1+\sigma)^d\xi^{-1},$$
$$\tau_{D,d}(f)=\sup_S |Df|(1+\sigma)^d\omega.$$

We now define the Schwartz space after Harish-Chandra [2].

Definition 1. *Let $\mathcal{C}(S)$(resp. $\mathcal{S}(S)$) denote the space of all $f\in C^\infty(S)$ such that $\nu_{D,d}(f)<+\infty$ (resp. $\tau_{D,d}(f)<+\infty$) for all $D\in\boldsymbol{D}(S)$ and integers $d\geqq 0$.*