225. *Results Related to Closed Images of M-Spaces.* II

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This is a continuation of our paper [4]. Supplements to the results in [4] will be given, as well as a new proof and slight extension of a theorem of Nagata [5]. We use the same notation and terminology as in our previous paper, in particular, all spaces are assumed to be T_1-spaces.

4. Supplements and consequences of Theorem 3.1.

Proposition 4.1. *Let Y be a regular space such that conditions (a) and (b) of Theorem 3.1 are satisfied. Then the M-space X for which a closed continuous map $f: X \to Y$ exists may be chosen to be normal or paracompact according as Y is normal or paracompact.*

Proof. Let X be the space constructed in the proof of [4, Theorem 3.1]. First note that X is closed in $B \times Y$. To show this, assume $(\alpha, y) \notin X$. Then $y \notin \bigcap_{i=1}^{\infty} F_{i\alpha_i}$, with $\alpha = (\alpha_1, \alpha_2, \cdots)$. Hence there exists an $m \in N$ such that

$$y \cap \left[\bigcap_{i=1}^{m} F_{i\alpha_i}\right] = \emptyset.$$

If we take $V(y)$ as $Y - \bigcap_{i=1}^{m} F_{i\alpha_i}$, we have

$$[B(\alpha_1, \cdots, \alpha_m) \times V(y)] \cap X = \emptyset,$$

so X is closed in $B \times Y$.

Now Y is the closed image of an M-space X. Since X is a P-space by Morita [2, Theorem 6.3], so is Y by [2, Theorem 3.3]. Hence if Y is normal, $B \times Y$ is normal [2, Theorem 4.1]. Thus X is normal. On the other hand, if Y is paracompact, so is $B \times Y$. Then X is paracompact.

Remark. Note that if $\bigcap_{i=1}^{\infty} F_{i\alpha_i}$ is compact for any q-sequence $\{F_{i\alpha_i}\}$, then the map $\varphi: X \to B$ is perfect and hence X is paracompact.

Definition 4.2. A space X is said to be an *M^*-space* if and only if X has a sequence $\{\mathcal{F}_n : n \in N\}$ of locally finite closed covers satisfying condition (1) below:

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