217. *Markov Semigroups with Simplest Interaction.* I

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(Comm. by Kunihiko KODAIRA, M. J. A., Oct. 12, 1971)

In this paper we will give a formulation of semigroups with simplest interaction (briefly, interaction semigroups) in an analogous way to that of branching semigroups. The most important examples and models of this class of semigroups are the solutions of spatially homogeneous Boltzmann equations with finite cross-sections, which will be proved in Part II to be the reversed processes of some branching processes. The key idea to reduce such a nonlinear evolution equation to a linear one is based on the fact that the former equation is the equation on the maximal ideal spaces of a certain commutative Banach algebras. Another example is Burgers' equation which can be solved along this idea.

§0. Notations.

0.0. Let Q be a compact Hausdorff space with a countable basis, Q^n the n-fold direct product of the space, Q_n the n-fold symmetric direct product for each positive integer n, $Q^*=\bigcup_{n\geqq 1} Q^n$ and $Q_*=\bigcup_{n\geqq 1} Q_n$. All of these spaces are compact except Q^* and Q_* which are locally compact. For any topological space X, $\mathcal{C}(X)$ denotes the totality of continuous functions on X with usual topology and $\mathcal{M}(X)$ the totality of Borel measures on X with finite mass for each compact subset in X.

0.1. For each $n\geqq 1$, R_n is the restriction map from $\mathcal{C}(Q^*)$ onto $\mathcal{C}(Q^n)$ and I_n is the operator from $\mathcal{C}(Q^n)$ into $\mathcal{C}(Q^*)$ defined by

$$I_n f=\begin{cases} f & \text{on} \quad Q^n \\ 0 & \text{off} \quad Q^n \end{cases} \qquad \text{for} \quad f\in \mathcal{C}(Q^n).$$

The image of I_n is in $\mathcal{C}_0(Q^*)$ the totallity of elements is $\mathcal{C}(Q^*)$ which tend to zero at infinity and $\sum I_n R_n$ is the identity on $\mathcal{C}(Q^*)$ and $\mathcal{C}_0(Q^*)$. S_n is the symmetrizing operator on $\mathcal{C}(Q^n)$ i.e.

$$S_n f(x_1,\cdots,x_n)=\frac{1}{n!}\sum_{\sigma} f(x_{\sigma(1)},\cdots,x_{\sigma(n)})$$
$$(f\in\mathcal{C}(Q^n),\ (x_1,\cdots,x_n)\in Q^n)$$

where the summation is taken over all the permutations of $\{1,\cdots,n\}$. The image of $\mathcal{C}(Q^n)$ by S_n may naturally be identified with the space $\mathcal{C}(Q_n)$, and the images of $\mathcal{C}(Q^*)$ and $\mathcal{C}_0(Q^*)$ by $S=\sum_{n\geqq 1} I_n S_n R_n$ with the spaces $\mathcal{C}(Q_*)$ and $\mathcal{C}_0(Q_*)$, respectively.

The following operator M from $\mathcal{C}(Q)$ into $\mathcal{C}(Q^*)$ is an essential one.