127. *The Order of Fourier Coefficients of Function of Higher Variation*

By Rafat N. SIDDIQI
Department of Mathematics, University of Sherbrooke, Sherbrooke, Quebec, Canada

(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 12, 1972)

This paper contains two theorems. First we estimate the order of Fourier coefficients of function of Wiener's class V_p which is strictly larger class than that of the class of functions of bounded variation. We have been able to find out the best constant which terms out to be $V_p(f)\pi^{-1}2^{1/q}$ in our case. The second theorem concerns about how many Fourier coefficients can have exactly the order $n^{-1/p}$.

1. Let f be a real valued 2π-periodic function defined on $[0,2\pi]$ and let $0=t_0\le t_1\le t_2\le\cdots\le t_n=2\pi$ be a partition of $[0,2\pi]$. We write, for $1\le p<\infty$,

$$V_p(f)=\sup\left\{\sum_{i=1}^{n}|f(t_i)-f(t_{i-1})|^p\right\}^{1/p} \tag{1}$$

where sup is taken over all partitions of $[0,2\pi]$. We say that a function f belongs to V_p or f is the function of p-th variation if $V_p(f)<\infty$. In terms of Wiener [5] we denote the class of all 2π-periodic functions of p-th variation on the segment $[0,2\pi]$ by V_p. We call $V_p(f)$ the p-th total variation of f. It can easily be verified that

$$V_p\subset V_q \qquad (1\le p<q<\infty) \tag{2}$$

is a strict inclusion. For $p=1$, V_1 is the class of functions of bounded variation. Let

$$\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx) \tag{3}$$

be a Fourier series of f. In the case V_1 the following theorem is well known [1] (see also [7]).

Theorem A. *If f belongs to V_1 then*

$$|a_n|\le V_1(f)(\pi n)^{-1}; \qquad |b_n|\le V_1(f)(\pi n)^{-1} \tag{4}$$

for all $n>1$, where $V_1(f)$ is the first total variation of f over $[0,2\pi]$.

Recently M. Taibleson [3] has proved a weaker form of Theorem A by an elementary method (see also [1] page 210). M. and S. Izumi [2] have given another elementary proof of Theorem A with the best constant $V_1(f)\pi^{-1}$ in (4). We extend Theorem A in the following way.

Theorem 1. *If f belongs to V_p $(1\le p<\infty)$ then*

$$\begin{cases}|a_n|\le V_p(f)\pi^{-1}2^{1/q}n^{-1/p}\,;\\ |b_n|\le V_p(f)\pi^{-1}2^{1/q}n^{-1/p}\end{cases} \tag{5}$$