16. *Cohomology of Lie Algebras over a Manifold*

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In 1970 M. V. Losik [3] has generalized the de Rham complex in the higher order jet spaces and determined its cohomology group completely. Immediately later, Gelfand and Fuks [2] have given an alternative proof to Losik's result from the viewpoint of their general theory concerning the cohomology of vector fields. Actually, they have not only reformulated and extended Losik's result in terms of differential forms, but also established an interesting relation between a representation of the general linear group and the one of the vector fields induced from it; the latter representation depends essentially on the first jet of the tangent bundle. In view of the cohomology theory of Lie algebras, from these representations canonically arise two complexes: one is associated to the Lie algebra of formal vector fields without constant terms and the other is the Lie algebra of vector fields. Gelfand and Fuks have further clarified in the cited paper that the cohomology groups of these two complexes stand in close relation connected by some spectral sequence.

We shall here generalize their results to the case where the representations are concerned with the higher ordre jet of the tangent bundle. Moreover, we shall formulate and prove the finite dimensionality of the cohomology groups of vector fields associated to these representations in a considerably general form.

1. Let $\mathfrak{a}_n$ be the Lie algebra of formal vector fields with n indeterminates. That is, $\mathfrak{a}_n$ consists of the elements with the form $\xi=\sum_{\mu=1}^{n}\xi^{\mu}(x_1,\cdots,x_n)\partial/\partial x_{\mu}$, $\xi^{\mu}\in \boldsymbol{R}[[x_1,\cdots,x_n]]$, and has the bracket rule induced from the usual differentiation. We consider $\mathfrak{a}_n$ as a topological Lie algebra where $\mathfrak{a}_n$ is endowed with the Krull topology. Let $\mathfrak{m}$ be the maximal ideal of $\boldsymbol{R}[[x_1,\cdots,x_n]]$ and set $L_k=\mathfrak{m}^{k+1}\mathfrak{a}_n$. Then $\mathfrak{a}_n$ is a simple Lie algebra and each L_k $(k=0,1,2,\cdots)$ becomes an ideal of L_0. Moreover, we have $L_0/L_1\cong\mathfrak{gl}(n;\boldsymbol{R})$, which is in turn obtained from a splitting $L_0=\mathfrak{gl}(n;\boldsymbol{R})\oplus L_1$. Let V be a finite-dimensional vector space over $\boldsymbol{R}$. Let $C^p(L_0,V)$ be the space consisting of the continuous alternating p-linear maps from L_0 to V. Then we have

$$C^p(L_0,V)=\varinjlim C_k^p(L_0,V),$$

where $C_k^p(L_0,V)$ denotes the subspace of $C^p(L_0,V)$, the elements of which