15. On the Structure of Algebraic Systems

By Tsuyoshi FUJIWARA
Department of Mathematics, Yamaguchi University
(Comm. by K. SHODA, M.J.A., Feb. 12, 1954)

The structure of an algebraic system A has been discussed by K. Shoda[2)3)4)] under the following conditions:

SI. *A has a null-element.*

SII. *The subsystem generated by any two normal subsystems of A is normal in A.*

SIII. *The meromorphism of any two algebraic systems which are homomorphic to A is always class-meromorphism.*

(SII *and* SIII *are assumed for any subsystem of* A.)

G. Birkhoff has introduced in his book[1)] the following condition which is equivalent to SIII: *all congruences on A are permutable.*

In the present paper we shall give a new definition of normal subsystems, and study on the normal subsystems and the congruences of an algebraic system A (§1). Moreover under weaker conditions than SII, SIII (§2), we shall discuss the Jordan-Hölder-Schreier theorem (§3) and the Remak-Schmidt-Ore theorem for A (§4).

§1. Normal Subsystems and Congruences. Throughout this paper we put the following conditions on the algebraic system A to keep out the complication.

0. *All compositions are binary and single valued, moreover any two elements may be composable by any composition.*

I. *A has a null-element* e ($e\alpha e=e$ *for any composition* α).

A subset B of A is called a *subsystem* if B is closed under any composition of A and contains e.

Let $f(\xi_1, \dots, \xi_n)$ be a polynomial by compositions of A. In the following $f(X, x_2, \dots, x_n)$ denotes the set $\{f(x, x_2, \dots, x_n) : x \in X\}$, where $X \subset A$, $x_2, \dots, x_n \in A$. Then $f(X, x_2, \dots, x_n)$ is of course a subset of A.

Definition 1. *A subset C is called a coset if and only if the following condition holds for any polynomial* $f(\xi_1, \dots, \xi_n)$ *and any elements* $x_2, \dots, x_n \in A$,

$$f(C, x_2, \dots, x_n) \frown C \neq \phi \quad \textit{implies} \quad f(C, x_2, \dots, x_n) \subset C.$$

A coset C is called a normal subsystem, when C forms a subsystem of A.

Theorem 1. *Any coset C is a residue class of a congruence and conversely.*