33. *Probabilities on Inheritance in Consanguineous Families. V*

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V. Mother-descendant combinations through a single consanguineous marriage

1. Mother-descendant combination immediate after a consanguineous marriage

Up to the last chapter, any consanguineous marriage has never been implicated. We now begin to attack the problems concerning a consanguineous marriage.

Let μth and νth descendants collaterally originated from a mother $A_{\alpha\beta}$ and her same spouse be married consanguineously and then originate themselves an nth descendant $A_{\xi\eta}$. Our present purpose is to determine the probability of combination $(A_{\alpha\beta};\ A_{\xi\eta})$ which will be designated by

$$\pi_{\mu\nu;n}(\alpha\beta;\ \xi\eta) \equiv \overline{A}_{\xi\eta}\kappa_{\mu\nu;n}(\alpha\beta;\ \xi\eta).$$

We distinguish *three systems* according to $\mu=\nu=1$, $\mu>1=\nu$ or $\mu=1<\nu$, and $\mu,\ \nu>1$. However, the final results for $\pi_{\mu\nu;n}$ will be, contrary to $\pi_{\mu\nu}$ discussed in III, unified into a unique expression for any pair of μ, ν with $\mu\geqq 1$, $\nu\geqq 1$.

We first deal with *the case* $n=1$. Its defining equation given by

$$\kappa_{\mu\nu;1}(\alpha\beta;\ \xi\eta) = \sum \kappa_{\mu\nu}(\alpha\beta;\ ab,\ cd)\varepsilon(ab,\ cd;\ \xi\eta)$$

leads to an expression

$$\kappa_{\mu\nu;1}(\alpha\beta;\ \xi\eta) = \overline{A}_{\xi\eta} + L_{\mu\nu}Q(\alpha\beta;\ \xi\eta) + 2^{-\lambda}T(\alpha\beta;\ \xi\eta),$$

where we put

$$L_{\mu\nu} = 2^{-\mu} + 2^{-\nu}, \qquad \lambda = \mu+\nu-1.$$

The values of the quantity defined by

$$T(\alpha\beta;\ \xi\eta) = 2\{\kappa_{11;1}(\alpha\beta;\ \xi\eta) - \kappa(\alpha\beta;\ \xi\eta)\}$$

are set out in the following lines:

$$\begin{aligned}
&T(ii;\ ii) = \tfrac{1}{4}(1-i)(2-i), && T(ii;\ ik) = -\tfrac{1}{2}k(2-i),\\
&T(ii;\ kk) = \tfrac{1}{4}k(1+k), && T(ii;\ hk) = \tfrac{1}{2}hk;\\
&T(ij;\ ii) = \tfrac{1}{8}(1-2i+2i^2), && T(ij;\ ij) = \tfrac{1}{4}(1-2i-2j+2ij),\\
&T(ij;\ ik) = -\tfrac{1}{2}k(1-i), && T(ij;\ kk) = \tfrac{1}{4}k(1+k),\\
&T(ij;\ hk) = \tfrac{1}{2}hk.
\end{aligned}$$

It can be shown that there hold the relations