35. On Symbolic Representation

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M. Morse and G. A. Hedlund have shown the method of symbolic dynamics and proved the interesting theorems in their papers [1], [2]. Those theorems seem as if they are independent of classical dynamics but they are indeed a new representation of classical interesting theorems

In this paper we shall prove the theorems of symbolic representation. These theorems are applicable to transitivity problem.

1. We consider a closed two-dimensional Riemannian manifold $\sum$ which is of genus $p\geqq 1$. The adding assumption is that no geodesic on $\sum$ has on it two mutually conjugate points.

When $p>1$ a convex domain S_0 in the unit circle regarded as the non-Euclidean plane Φ is bounded by a sequence

$$B_1^{-1}, A_1^{-1}, B_1, A_1, B_2^{-1}, A_2^{-1}, B_2, A_2, \dots, B_p^{-1}, A_p^{-1}, B_p, A_p,$$

of congruent segments of H-straight (H means hyperbolic) lines such that each pair of the successive H-lines forms an angle equal to $\frac{\pi}{2p}$. If we identify congruent points of conjugate sides of S_0, we get a closed orientable surface T of genus p with constant negative curvature.

Let $\tilde{B}_1, \tilde{A}_1, \tilde{B}_2, \tilde{A}_2, \dots, \tilde{B}_p, \tilde{A}_p$ be a set of geodesics which starts from and comes back to a point P of $\sum$ and every geodesic of the set be. homotopic to a curve of canonical section of $\sum$. Then we can select those geodesics so as to be independent each other if we choose P suitably.

We map $\sum$ topologically on T and $\tilde{A}_i, \tilde{B}_i$ on A_i, B_i respectively and denote this map f.

When $p=1$ a convex domain S_0 in Euclidean plane Φ is bounded by a sequence

$$B_1^{-1}, A_1^{-1}, B_1, A_1,$$

of congruent segments of E-straight lines (E means Euclidean) such that each pair of successive H-lines forms an angle equal to $\frac{\pi}{2}$ If we identify congruent points of conjugate sides of S_0, we get a closed orientable surface T of genus 1 with vanishing curvature.

Let $\tilde{B}_1, \tilde{A}_1$ be geodesics which start from and come back to a