137. Probabilities on Inheritance in Consanguineous Families. XI

By Yûsaku KOMATU and Han NISHIMIYA
Department of Mathematics, Tokyo Institute of Technology
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VIII. Combinations through the most extreme consanguineous marriages

5. Distributions after successive consanguineous marriages

We have derived in §3 the probability of parent-descendant combination. Elimination of a type of parent leads to a corresponding *distribution of genotypes in a generation of descendant*. Namely, we have, for any $n\geqq 1$, a relation

$$\overline{A}_{(11;0)_{t-1}|n}(\xi\eta)=\sum \overline{A}_{ab}\mathfrak{k}_{t-1|n}(ab;\xi\eta).$$

In case $n=1$, we get, by actual computation,

$$\overline{A}_{(11;0)_{t-1}|1}(ii)=i-i(1-i)\boldsymbol{R}\frac{5+3\sqrt{5}}{5}\omega^t,$$

$$\overline{A}_{(11;0)_{t-1}|1}(ij)=\qquad 2ij\boldsymbol{R}\frac{5+3\sqrt{5}}{5}\omega^t.$$

The result shows that the distribution *deviates* from ordinary one. More precisely, there hold the relations

$$\overline{A}_{(11;0)_{t-1}|1}(\xi\eta)-\overline{A}_{\xi\eta}=2R(\xi\eta)\left(1-\boldsymbol{R}\frac{5+3\sqrt{5}}{5}\omega^t\right)$$

with $R(ii)=\frac{1}{2}i(1-i)$ and $R(ij)=-ij$. Namely, *any homozygous type increases while any heterozygous one decreases.* For instance, the values of the factor $1-\boldsymbol{R}(1+3\sqrt{5}/5)\omega^t$ are equal to 11/20, 23/40, 27/40, 117/160 etc. for $t=2$, 3, 4, 5 etc., respectively.

Though there exists a deviation in the first generation, it *vanishes out* soon in the next generation. In fact, as shown in §3, we have an identity $\mathfrak{k}_{t-1|n}=\kappa_n$ for any $n>1$, whence readily follows

$$\overline{A}_{(11;0)_{t-1}|n}(\xi\eta)=\overline{A}_{\xi\eta}.$$

By the way, it would be noted that the frequency of gene A_i in the first generation is given by

$$\overline{A}_{(11;0)_{t-1}|1}(ii)+\frac{1}{2}\sum_{b\neq i}\overline{A}_{(11;0)_{t-1}|1}(ib)=i.$$

Consequently, the random matings *within* the generation produce also the distribution coincident with the original one, i.e. $\overline{A}_{ii}=i^2$, $\overline{A}_{ij}=2ij$.