123. *Relations between Harmonic Dimensions*

By Zenjiro KURAMOCHI
Mathematical Institute, Osaka University

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M. Ozawa proposed the following problem:[1] Let F be a null-boundary Riemann surface with one ideal component and D be a non compact domain which has a finite number of analytic curves as its relative boundary. Denote by $\overset{**}{\dim} D$ the number of linearly independent generalized Green's functions. (See the definition given below.) Let F_0 be a compact disc which has no common point with D. Then we have the relation:

$$\overset{**}{\dim} D \leqq \dim (F-F_0)?$$

It is the purpose of this article to give a solution to the problem.

Let F be an abstract Riemann surface, $\{F_n\}$ an exhaustion of F and D a non compact domain of F, whose relative boundary ∂D[2] consists of at most enumerable number of analytic curves clustering nowhere in F. Let $\{p_i\}$ be a sequence of points in D, such that $\{p_i\}$ converges to the boundary of F, and let $G(z, p_i)$ be the Green's function of D with pole at p_i. Take a subsequence of $\{G(z, p_i)\}$ which converges uniformly to a non-constant function $G(z, \{p'_i\})$ which we call *generalized Green's function*. Denote by F_0 a compact disc which has no common point with D and let $G_{F-F_0}(z, p_0)$ be the Green's function of $F-F_0$, where p_0 is an inner point of D. In this case, it is clear that $\infty > \overline{\lim_i} G_{F-F_0}(p_i, p_0) \geqq \lim_i G(p_i, p_0) > 0$. $\infty > \overline{\lim_i} G_{F-F_0}(p'_i, z) = G_{F-F_0}(z, \{p'_i\}) \geqq G(z, \{p'_i\})$ for every point z, whence $G(z, \{p'_i\})$ is finite in $D \cap F_n$ $(n=1, 2, 3, \ldots)$. Put $D^N(\{p'_i\}) = \mathcal{E}\{z; G(z, \{p'_i\}) \geqq N\}$. We denote by $\hat{D}^N(p_0)$ the symmetric surface of $D^N(p_0)(=\mathcal{E}\{z; G(z, p_0) \geqq N\})$ with respect to $\partial D^N(p_0)$. Then $D^N(p_0)+\hat{D}^N(p_0)$ is a null-boundary[3] Riemann surface.

Lemma.
$$\int_{\partial D^N(\{p'_i\})} \frac{\partial G(z, \{p'_i\})}{\partial n} ds = \delta(\{p'_i\}) \leqq 2\pi.$$

Proof. Denote by $\{D_n\}$ the exhaustion of D. Since $G(z, p_i)$ is bounded outside a neighbourhood v of p_i, we have $D_{D^N(p_i)-v}(G(z, p)) < \infty$

1) At the annual meeting of the Mathematical Society of Japan held on the 30th of May, 1954. M. Ozawa: On harmonic dimensions I and II, to appear in Kdōai Mathematical Seminar Reports.

2) In this article we denote by ∂G the relative boundary of G with respect to F.

3) Z. Kuramochi: Harmonic measures and capacity of a subset of the ideal boundary of abstract Riemann surface, to appear in the Proceedings of the Japan Academy.