122. A Note on *f*-completeness

By Shouro KASAHARA
Kobe University

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In a recent paper [2], A. W. Ingleton introduced a concept, *spherically completeness*, which is important for the extension of continuous linear mappings of a non-Archimedean normed space into another one. For a locally flat topological linear space whose topology is defined by a family of non-Archimedean semi-norms, the author has given a concept, *f-completeness* [3], on the extension property.

It is our purpose in this note to prove some conspicuous properties on f-completeness.

Throughout this note, we will denote by K a non-Archimedean valued field of which the valuation v is non-trivial, and assume that the locally flat linear spaces have the same K as the underlying field of scalars, and moreover by *f-complete space* we shall mean a locally flat linear space which is f-complete with respect to each of the non-Archimedean semi-norms defining the topology.

Let (E_ι) be a family of locally flat linear spaces, and let us consider the product space $F=\Pi_\iota E_\iota$, and denote by f_ι the projection of F to E_ι. Then it is clear that the topology of the linear space F is defined by the family of non-Archimedean semi-norms $p_\alpha \circ f_\iota$, where for any ι, p_α runs over the family of non-Archimedean semi-norms defining the topology of E_ι. That is, *the product space of a family of locally flat linear spaces is locally flat.*

The following proposition can be readily verified.

Proposition 1. (a) *The product of a family of f-complete spaces is also f-complete.* (b) *If W is a closed subspace*[1] *of an f-complete space E, then the quotient space E/W is f-complete.*

The part (a) of the proposition is clear.

Let p^* be the non-Archimedean semi-norm of E/W corresponding to a non-Archimedean semi-norm p of the space E. Then the inverse image of any p^*-flat variety in E/W by the canonical mapping π of E onto E/W is a p-flat variety in E, and hence the part (b) is clear.

Proposition 2. *Let W be an f-complete subspace of a Hausdorff linear space E; then W admits a topological supplement,*[2] *and is therefore closed.*

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1) In this note "subspace" always means "linear subspace".
2) Cf. (1) p. 16.