# 113. *Note on Deformation Retract*

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1. The main object of this note is to study a mapping which has a torus as the image space. The methods of the paper are strongly influenced by Spanier's proofs [5].

2. In this section we prepare some definitions and lemmas known in Spanier's paper on Borsuk's cohomotopy groups [5], [2].

Let $\mathfrak{X}$ denote the space of a sequence of real numbers $y=(y_i)$ $(i=1,2,\dots)$ which are finitely non-zero (i.e. $y_i=0$ except for a finite set of integers $i$). $\mathfrak{X}$ is metrized by

$$\operatorname{dist}(y,y')=(\sum_i (y_i-y'_i)^2)^{\frac{1}{2}}.$$

**Definition 2.1.** The sets below are defined by the corresponding condition on the right:

$$S^n=\{y\in\mathfrak{X}\mid y_i=0 \quad \text{for} \quad i>n+1 \quad \text{and} \sum_{1\leq i\leq n+1} y_i^2=1\},$$
$$E^{n+1}=\{y\in\mathfrak{X}\mid y_i=0 \quad \text{for} \quad i>n+1 \quad \text{and} \sum_{1\leq i\leq n+1} y_i^2\leq 1\},$$
$$E^n_+=\{y\in S^n\mid y_{n+1}\geqq 0\},$$
$$E^n_-=\{y\in S^n\mid y_{n+1}\leqq 0\},$$
$$E^0_+=p=(1,0,\dots,0,\dots),$$
$$E^0_-=\bar{p}=(-1,0,\dots,0,\dots),$$
$$T^{2n}=S^n\times S^n, q=p\times p, \bar{q}=\bar{p}\times\bar{p} \qquad (\text{for} \quad n\geqq 1).$$

**Lemma 2.2.** Let $A$ be a deformation retract [4] of a compact space $X$ and let $f\colon (X,A)\to(Y,B)$ be a map of $(X,A)$ onto $(Y,B)$, which maps $X-A$ homeomorphically onto $Y-B$. Then $B$ is a deformation retract of $Y$.

**Lemma 2.3.** Let $(X,A)$ be a compact pair with $\dim(X-A)\leqq n$. If $F$ is any closed subset of $X\times I-A\times I$, $\dim F\leqq n+1$.

**Definition 2.4.** Let $f\colon (X,A)\to(Y\times Y,(y,y))$. A map

$$F\colon \quad (X\times I, A\times I)\to(Y\times Y,(y,y))$$

will be called a normalizing homotopy for $f$, if

$$\left.\begin{array}{l} F(x,0)=f(x) \\ F(x,1)\in(Y\times y)\cup(y\times Y)\end{array}\right\} \text{ for all } \quad x\in X.$$

The map $f'\colon (X,A)\to[(Y\times y)\cup(y\times Y),(y,y)]$ defined by $f'(x)=F(x,1)$ is called a normalization of $f$.

In the following $Y\vee Y$ will denote the space $(Y\times y)\cup(y\times Y)$. Let

$$\Omega\colon \quad [Y\vee Y,(y,y)]\to(Y,y)$$

be defined by