199. Groups of Isometries of Pseudo-Hermitian Spaces. I

By Shigeru ISHIHARA
Tokyo Metropolitan University

(Comm. by K. KUNUGI, M.J.A., Dec. 13, 1954)

Recently, Prof. K. Yano [6] has proved beautiful theorems about groups of isometries of n-dimensional Riemannian spaces. We shall study groups of isometries of a pseudo-Hermitian space, by an analogous method.

1. *Preliminary.* Let M be a pseudo-Hermitian space of $2n$ dimensions of class C^4. Then there exists such a tensor field φ^i_j of type (1,1) that

$$\varphi^i_a \varphi^a_j = -\delta^i_j, \qquad g_{ab}\varphi^a_i \varphi^b_j = g_{ij},$$

where g_{ij} is the metric tensor of the space M, and φ^i_j and g_{ij} are of class C^3. If we put

$$\varphi_{ij} = g_{ia}\varphi^a_j,$$

then φ_{ij} is a skew-symmetric tensor by virtue of the relation $\varphi^i_a\varphi^a_j = -\delta^i_j$. When the tensor φ_{ij} is covariant constant, the space M is pseudo-Kählerian.

Let G be a group of isometries of M onto itself and φ^i_j be invariant by G. For brevity, we call G a *group of Hermitian isometries*. If the group G is transitive on M, the space M is called a *homogeneous pseudo-Hermitian space* by definition. Furthermore, if the Riemannian metric g_{ij} of the homogeneous pseudo-Hermitian space M is pseudo-Kählerian, then M is called a *homogeneous pseudo-Kählerian space*.

Let G be a group of Hermitian isometries of a pseudo-Hermitian space M and H the subgroup of G, each transformation of which fixes a given point O of M. That is to say, H is the group of isotropy at the point $O \in M$. Then the subgroup H is isomorphic to a subgroup H' of the unitary group $U(n)$ in n complex variables and H operates on the tangent space of M at the point O in the same manner as the real representation of H' which operates on the $2n$-dimensional real vector space. Throughout this paper, we assume that the group G is always effective on M, and that the group G and the space M are both connected. Moreover, for brevity, it is supposed that the subgroup H of isotropy is connected.

2. *Subgroups of $U(n)$ of dimension $r \geqq n^2-2n+2$.* The following theorem is proved by using the theorems due to D. Montgomery and H. Samelson [3].