198. *Some Properties of Hypernormal Spaces*

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E. Hewitt (3) has defined a new class of abstract space called *hypernormal space*. Further results on hypernormal spaces have been obtained by M. Katětov (6). This note is concerned with a consideration of hypernormal spaces.

All spaces considered are Hausdorff or T_2 spaces.

Definition. A space S is called *hypernormal*, if, for any two separated subsets A, B of S, there are two open sets G, H such that $G\supset A$, $H\supset B$ and $\overline{G}\frown\overline{H}=0$.

We shall first prove the following

Theorem 1. For a Hausdorff space S, the following statements are equivalent.

(1) *S is hypernormal,*

(2) *If A and B are separated, there is a continuous function f on S such that $f(x)=0$ for each $x\in A$ and $f(x)=1$ for each $x\in B$.*

(3) *If A is any subset of S, and f is a bounded continuous function on A, f may be extended to continuous on S.*

In the terminology of E. Čech (1) and E. Hewitt (4), the statement (2) is that any two separated set A, B of S are always *completely separated.*

The statement (3) is essentially due to M. Katětov (6).

Proof. $(1)\rightarrow(2)$

Let A, B be separated sets of S, then there are two open sets G, H such that $G\supset A$, $H\supset B$ and $\overline{G}\frown\overline{H}=0$. Since any hypernormal space is normal, there is a continuous function f on S such that $f(x)=0$ on G and $f(x)=1$ on H. Thus A, B are completely separated.

$(2)\rightarrow(3)$

We can suppose that f on A has the values in $[-1, 1]$. Let $f_0=f$. We shall define inductively f_n and φ_n. We suppose that f_n are defined. Let

$$A_n=\left\{x\in S\,|\,f_n(x)\leq\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^n\right\}$$

$$B_n=\left\{x\in S\,|\,f_n(x)\geq\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^n\right\}$$

then A_n, B_n are separated. The functions