54. Evans-Selberg's Theorem on Abstract Riemann Surfaces with Positive Boundaries. II

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Our $N_{V_m(p)}(p,q)$ is increasing with respect to m. We define the value of $N(z,q)$ at a minimal point p by $\lim_{m\to M'} N_{V_m(p)}(p,q)$ denoted by $N(p,q)$. If p or q belongs to R, this definition is equivalent to that defined before.

If $V_m(p)$ is not regular, we define $N_{V_m(p)}(p,q)$ by $\lim_{m'\to m} N_{V_{m'}(p)}(p,q)$, where $m'<m$ and $V_{m'}(p)$ is regular. In the case when $V_m(p)$ is regular, it is proved that $\lim_{m'\to m} N_{V_{m'}(p)}(p,q)=N_{V_m(p)}(p,q)$, hence we can define $N_{V_m(p)}(p,q)$ for every $m<\sup_{z\in R} N(z,p)=M'$. As in case of a Riemann surface with a null-boundary, we can prove the following

Theorem 10. 1) *$N(z,q)$ $(q\in \overline{R})$ is δ-lower semicontinuous in $R+B_1$.*

2) *$N(z,q)$ is superharmonic in weak sense at every point of $R+B_1$.*

3) *If p and q are in $R+B_1$, then $N(p,q)=N(q,p)$.*

Till now $N(z,q)$ $(q\in\overline{R})$ is defined only on $R+B_1$. Next we define $N(z,q)$ at points belonging to B_0. If $p\in B_0$, $N(z,p)=\int_{B_1} N(z,p_\alpha)d\mu(p_\alpha)$ $(p_\alpha\in B_1)$ by Theorem 8. Although the uniqueness of this mass distribution is not proved by the present author, the value of $N(z,q)$ in $R+B_1$ is uniquely determined. On the other hand, by 3), for $q\in B_1$, $N(p_\alpha,q)=N(q,p_\alpha)$. Hence it is quite natural to define the value of $N(z,q)$ at $p\in B_0$ by $\int N(p_\alpha,q)d\mu(p_\alpha)$. Evidently by 3), in such definition, we have $N(q,p)=N(p,q)$, where the term of the right hand side does not depend on a particular distribution but on the behaviour of $N(z,q)$, because $N(p,q)=\lim_{m\to M'} N_{V_m(p)}(p,q)$ and $N_{V_m(p)}(p,q)$ is defined by the value of $N(z,q)$ on $\partial V_m(p)$. As for the behaviour of $N(z,q)$ $(q\in\overline{R})$, we have the following

Theorem 11. 1) *If $q\in R+B_1$, then $N(p,q)=N(q,p)$ for $p\in\overline{R}$.*

2) *If $q\in\overline{R}$ and $p\in R+B_1$, then $N(p,q)=\int N(p,q_\alpha)d\mu(q_\alpha)$, where $N(z,q)=\int N(z,q_\alpha)d\mu(q_\alpha)$.*

3) *$N(z,q)$ $(q\in\overline{R})$ is δ-lower semicontinuous in $\overline{R}$.*