88. *Note on Algebras of Strongly Unbounded Representation Type*[1)]

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§1. Let A be an associative algebra with a unit element over an algebraically closed field k and $g_A(d)$ be the number of inequivalent indecomposable representations of A of degree d where d is a positive integer. Then if A has indecomposable representations of arbitrary high degrees and $g_A(d)=\infty$ for an infinite number of integers d, A is said to be of *strongly unbounded representation type*. In a paper [1] James P. Jans proved that the following four conditions are sufficient for an algebra to be of strongly unbounded representation type:

(*1*) *L_A, the two-sided ideal lattice, is infinite.*

(*2*) *For any i and any two-sided ideal A_0 in N (N is the radical of A) e_iA_0 (A_0e_i) has more than three covers in e_iN (Ne_i) where A' is said to be the cover of e_iA if $A'\supset e_iA_0$ and $A'\supset B\supseteq e_iA_0$ implies $B=e_iA_0$.*

(*3*) *The graph $G(A_0)$ associated with any two-sided ideal $A_0\subset N$ is a cycle where the graph $G(A_0)$ is such a set $\{P_1, P_1 \& P_2, P_2, P_2 \& P_3, P_3, \cdots, P_{n-1}, P_{n-1} \& P_n, P_n\}$[2)] that $P_i \& P_j$ holds if $e_iA'e_j$ covers $e_iA_0e_j$ for some cover A' of A_0 and $G(A_0)$ is said to be the cycle if $\{G(A_0), G(A_0)\}$ is also a graph.*

(*4*) *The graph $G(A_0)$ associated with any two-sided ideal $A_0\subset N$ branches at each end where $G_1(A_0)$ is said to extend $G_2(A_0)$ at the right end if $\{G_2(A_0), G_1(A_0)\}$ is the graph and $G(A_0)$ is said to branch at one end if it is extended by at least two distinct graphs at one end.*

Now in this paper we shall prove that the following two conditions are also sufficient for an algebra to be of strongly unbounded representation type:

(*5*) *The graph $G(A_0)$ associated with any two-sided ideal $A_0\subset N$ is* $\left\{\begin{matrix} P_{r_2}, P_{k_1} \& P_{r_2}, P_{k_1}, P_{k_1} \& P_{r_1}, \\ P_{r_3}, P_{k_2} \& P_{r_3}, P_{k_2}, P_{k_2} \& P_{r_1}, \end{matrix} P_{r_1}, P_{k_3} \& P_{r_1}, P_{k_3}, P_{k_3} \& P_{r_4}, P_{r_4}\right\}$.

(*6*) *The graph $G(A_0)$ is* $\left\{P_{k_5}, P_{k_5} \& P_{j_4}, P_{j_4}, P_{k_4} \& P_{j_4}, P_{k_4}, P_{k_4} \& P_{j_3}, P_{j_3}, P_{k_3} \& P_{j_3}, P_{k_3}, \begin{matrix} P_{k_3} \& P_{j_2}, \\ P_{k_2}, P_{k_2} \& P_{j_2}, \end{matrix} P_{j_2}, P_{k_1} \& P_{j_2}, P_{k_1}, P_{k_1} \& P_{j_1}, P_{j_1}\right\}$.

1) James P. Jans [1].

2) $P_1, P_2, \cdots, P_n$ mean vertices, and "P_i & P_j" means that "P_i and P_j are connected by an (oriented) edge".