

102. Note on Algebras of Bounded Representation Type

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1. Let A be an associative algebra with a unit element over a field k , N be the radical of A and $g_A(d)$ be the number of inequivalent indecomposable representations of A of degree d where d is an integer. Now A is said to be of *bounded representation type* if there exists an integer d_0 such that $g_A(d)=0$ for all $d \geq d_0$ and A is said to be of *finite representation type* if $\sum_d g_A(d)$ is finite.¹⁾

Concerning these classes of algebras of bounded type and finite type, Professor Brauer and Professor Thrall conjectured that these two classes are identical.²⁾ This conjecture is not yet proved, but now we shall prove it in a special case where $N^2=0$ and k is algebraically closed.

2. By the same way as [2] we may assume that A is the basic algebra. Then for this purpose we have only to prove that arbitrary two representations by indecomposable A -left modules $\mathfrak{M}_1 = \sum_{i=1}^r \sum_{\lambda_i=1}^{s_i} A e_i m_{i,\lambda_i}$ and $\mathfrak{M}_2 = \sum_{i=1}^r \sum_{\lambda_i=1}^{s_i} A e_i m'_{i,\lambda_i}$ which have the same type are equivalent. For the number of such A -left modules with different types is finite.³⁾

Now from the proof of the main theorem of [2], we may consider about the following three cases:

(a) $\{Ne_1, \dots, Ne_r\}$ is such a chain that Ne_r is the direct sum of three simple components.

(b) $\{Ne_1, Ne_2, Ne_3\}$ is such a chain that Ne_2 is the direct sum of three simple components.

(c) $\{Ne_1, Ne_2, Ne_3, Ne_4\}$ is such a chain that Ne_2 is the direct sum of three simple components and Ne_1 and Ne_4 are simple.

(i) First suppose that $\{Ne_1, \dots, Ne_r\}$ is such a chain as (a). Then an arbitrary indecomposable representation by $\mathfrak{M} = \sum_i \sum_{\lambda_i} A e_i m_{i,\lambda_i}$ has the following form:

1) James P. Jans [1].

2) James P. Jans [1].

3) In this paper we use the results of [2] without proof.