147. *Probabilities on Inheritance in Consanguineous Families. XV*

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XII. Generalization to the case of several ancestors

1. Ancestors-descendant combination

In the preceding chapter[1] we have dealt with a mother-descendants combination in a consanguineous lineage of general nature and further with some related combinations which result from the former by means of simple procedures of elimination. However, every combination considered there has concerned, in principle, only one distinguished ancestor, while the number of descendants has been supposed one or two. It seems now plausible that the results may be generalized to the case of several ancestors, which will be discussed in the present chapter.

For the sake of brevity, we confine ourselves throughout the present chapter to a supposition that every generation-number between two consecutive critical positions in a lineage is, in principle, greater than unity, unless a contrary is stated.

We now consider an ancestors-descendant combination in a non-consanguineous lineage with an assigned number of distinguished ancestors. All the possible types of ancestors-descendant combination which corresponds to a fixed number D of distinguished ancestors will be classified into $\Phi(D)$ classes according to the topological structure of lineage.

Let a combination concerning a non-consanguineous lineage be given which consists of D distinguished ancestors $\mathbf{0}_\delta\ (\delta=1,\cdots,D)$ and a common descendant $\mathbf{1}$. The set of these ancestors will be designated abbreviatedly by

$$\mathfrak{a}_D\equiv(\mathbf{0}_1,\cdots,\mathbf{0}_D),$$

the order of the constituents being a matter of indifference. It is shown that the reduced probability of the combination consisting of these $D+1$ individuals is given by the formula

$$K^F(\mathfrak{a}_D;\mathbf{1})=\bar{A}_1+2\sum_{\mathbf{0}_\delta\in\mathfrak{a}_D}2^{-\mathbf{1}(\mathbf{0}_\delta)}Q(\mathbf{0}_\delta;\mathbf{1})\qquad(F=\mathrm{I},\mathrm{II},\cdots,\Phi(D)).^{2)}$$

1) For previous part of the paper cf. Proc. Japan Acad. **31** (1955), 570–574. Details of the present paper will be published soon in Bull. Tokyo Inst. Tech.

2) We designate, as before, by $\mathbf{1}(\mathbf{0}_\delta)$ the length of the path from $\mathbf{0}_\delta$ to $\mathbf{1}$.