128. *Ideal Theory of Semiring*

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Quite recently, some writers have considered a non-commutative lattice which is a generalisation of the notion of lattices and have shown that the theory of non-commutative lattices are very useful for the theoretic physics. On the other hand, any semirings we shall develop are considered as a extensive generalisation of a non-commutative case for distributive lattices. In this paper, we shall develop the ideal theory of a semiring[1] and consider a structure space of a semiring.

Let R be a semiring. Unless otherwise stated, the word *ideal* shall mean two-sided ideal.

Definition 1. An ideal P is *prime*, if and only if $AB\subset P$ for any two ideals A, B implies $A\subset P$ or $B\subset P$.

Definition 2. An ideal I is *irreducible*, if and only if $A\frown B=I$ for two ideals A, B implies $A=I$ or $B=I$.

Definition 3. An ideal S is *strongly irreducible*, if and only if $A\frown B\subset S$ for any two ideals A, B implies $A\subset S$ or $B\subset S$.

A notion of strongly irreducible ideals was introduced by L. Fuchs [5] who calls *primitive*. In his paper [2], R. L. Blair used a terminology strongly irreducible. We shall follow his terminology.

From $AB\subset A\frown B$ for any two ideals A, B, any prime ideals are strongly irreducible and any strongly irreducible ideals are irreducible.

Theorem 1. The following conditions are equivalent.

(1) *P is a prime ideal.*

(2) *If (a), (b) are principal ideals*[2] *and $(a)(b)\subset P$, then $a\in P$ or $b\in P$.*

(3) *$aRb\subset P$ implies $a\in P$ or $b\in P$.*

(4) *If I_1, I_2 are right ideals and $I_1I_2\subset P$, then $I_1\subset P$ or $I_2\subset P$.*

(5) *If I_1, I_2 are left ideals and $J_1J_2\subset P$, then $J_1\subset P$ or $J_2\subset P$.*

Theorem 1 was proved by N. H. McCoy [10] for the case of rings.

Proof. It is clear that (1) implies (2). To prove that (2) implies (3), let $aRb\subset P$, then $RaRbR\subset P$, and hence we have $(a)^2(b)^3\subset P$. This implies $a\in P$ or $b\in P$.

To prove that (3) implies (4), let $I_1I_2\subset P$ for right ideals I_1, I_2

1) For the detail of a semiring, see K. Iséki and Y. Miyanaga [8].

2) (a) denotes the principal two-sided ideal generated by a.