123. *Fourier Series. II. Order of Partial Sums*

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(Comm. by Z. SUETUNA, M.J.A., Oct. 12, 1956)

1. **Introduction.** Let $f(t)$ be an integrable function with period 2π and its Fourier series be

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt).$$

By $s_n(x)$ we denote the nth partial sum of the Fourier series. We put as usual $\varphi_x(u)=f(x+u)+f(x-u)$.

H. Lebesgue [1] has proved the following

Theorem 1. *If, for a fixed x,*

$$\int_0^t|\varphi_x(u)|\,du=o(t) \tag{1}$$

as $t\to 0$, then

$$s_n(x)=o(\log n). \tag{2}$$

S. Izumi [2] proved

Theorem 2. *The conditions*

$$(3)\quad \int_0^t\varphi_x(u)\,du=o(t),\qquad (4)\quad \int_0^t|\varphi_x(u)|\,du=O(t)\qquad (t\to 0)$$

do not imply (2) in general.

Then there arises the problem: What condition with (3) does (2) imply? As an answer of this problem we prove the following

Theorem 3. *If (3) holds and*

$$\int_0^t(f(\xi+u)-f(\xi-u))\,du=o(t)\qquad (t\to 0) \tag{5}$$

uniformly in ξ in a neighbourhood of x, then (2) holds.

This is proved by the same idea as in the proof of Theorem 7 in [4].

On the other hand O. Szász [3] proved that:

Theorem 4. *If*

$$\int_0^t|\varphi_x(u)|\,du=o\left(t\Big/\log\frac{1}{t}\right)\qquad (t\to 0), \tag{6}$$

then

$$s_n(x)=o(\log\log n). \tag{7}$$

Analogously as Theorems 2 and 3, we prove the following theorems.

Theorem 5. *The conditions*

$$\int_0^t\varphi_x(u)\,du=o\left(t\Big/\log\frac{1}{t}\right), \tag{8}$$