154. *Note on Mapping Spaces*

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1. The set of all continuous mappings of a topological space X into another topological space Y is turned into a topological space by the compact-open topology; this topology is defined by selecting as a sub-basis for the open sets the family of sets $T(K, G)$ where K ranges over all the compact sets of X and G ranges over all the open sets of Y and $T(K, G)$ denotes the set of all continuous mappings f of X into Y such that $f(K)\subset G$. As usual we write Y^X for the mapping space.

Let X, Y be Hausdorff spaces and let Z be a topological space. With any continuous mapping f of $X\times Y$ into Z there is associated a mapping f^* from Y to the mapping space Z^X by the formula

(1) $$[f^*(y)](x)=f(x, y).$$

The correspondence $f\to f^*$ defines a one-to-one mapping

(2) $$\theta: Z^{X\times Y}\to (Z^X)^Y.$$

R. H. Fox [2] proved that θ is onto if either (i) X is locally compact or (ii) X and Y satisfy the first axiom of countability. It will be shown below (Theorem 1) that θ is always a homeomorphism into. Therefore θ is a homeomorphism onto in the above two cases (i) and (ii).[1] However, the case in which X is a CW-complex in the sense of J. H. C. Whitehead [5] and Y is a compact Hausdorff space seems to be not treated in the literature in spite of its importance in applications. In this note we shall prove that θ is a homeomorphism onto in this case also (Theorem 4).[2] This result will be obtained from a more general theorem (Theorem 2).

2. A Hausdorff space X will be said to have the weak topology with respect to compact sets in the wider sense if a subset A of X such that $A\frown K$ is closed for every compact set K of X is necessarily closed.[3] As is proved in [4], a Hausdorff space X has the weak topology with respect to compact sets in the wider sense if and only if X is obtained as a decomposition space of a locally compact, paracompact Hausdorff space.

1) M. G. Barratt [1, p. 81] has stated without proof that θ is a homeomorphism onto in case (i) with Y arbitrary and in case (ii) with $Y=I$ (the closed unit interval).

2) Thus the track group $(P, Q)^m(X, x_0; x_0)$ in the sense of Barratt [1] is isomorphic to the m-th homotopy group of the mapping space $(X, x_0)^{(P,Q)}$ with the base point $P\to x_0$ in case P is a Hausdorff space which is locally compact or satisfies the first axiom of countability, or a CW-complex.

3) A T_1-space having the weak topology with respect to compact sets in the sense of [3] is nothing but a discrete space.