150. *Fourier Series. IV. Korevaar's Conjecture*

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1. J. Korevaar [1] has proved the following theorem.

Theorem 1. Let $f(x)$ be a periodic function with period 2π which is continuous except for a finite number of jump discontinuities and which belongs to the class Lip 1 on every subinterval where $f(x)$ is continuous. Then there is a constant A_1, depending on the Lipschitz constants, supremum of the absolute value of $f(x)$ and the set of jump points, such that for every n there is a trigonometrical polynomial

$$s_n(x)=\sum_{k=0}^{n}(a_k \cos kx+b_k \sin kx) \tag{1}$$

of order n, which satisfies

$$\int_{-\pi}^{\pi} |f(x)-s_n(x)|\,dx < A_1/n, \tag{2}$$

$$|a_k|<A_1/k, \quad |b_k|<A_1/k \quad (k=0,1,\cdots,n). \tag{3}$$

We can see, as an immediate consequence of a result due to A. Zygmund [2], that the left side of (2) can not be $o(1/n)$ as $n\to\infty$. In this connection J. Korevaar surmised the truth of the following

Conjecture. Let $f(x)$ be a periodic function with period 2π which is continuous except jump discontinuity at a point ξ and belongs to the class Lip 1 in the interval $(\xi, \xi+2\pi)$. Then there is a constant A_2 such that for every n and every trigonometrical polynomial $t_n(x)$ of order n

$$\int_{-\pi}^{\pi} |f(x)-t_n(x)|\,dx > A_2/n. \tag{4}$$

We shall here prove this.

2. Proof of the conjecture. Let

$$E_n(f;-\pi,\pi)=\min_{(t_n)}\int_{-\pi}^{\pi}|f(x)-t_n(x)|\,dx,$$

where the minimum is taken for all trigonometrical polynomial $t_n(x)$ of order n. It is sufficient to prove that

$$E_n(f;\,-\pi,\pi)\geqq A_2/n.$$

Evidently

$$E_n(f;\,-\pi,\pi)\geqq E_n(f;\,-\varepsilon\pi/n, \varepsilon\pi/n),$$

where ε is a positive number which will be determined later. If we put $t_n(x)=\sum_{k=0}^{n}(a_k \cos kx+b_k \sin kx)$, then