170. Note on Algebras of Strongly Unbounded Representation Type. II

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1. This paper is a continuation of our previous paper¹⁾ on algebras of strongly unbounded representation type. Let A be an algebra over an algebraically closed field k and $g_A(d)$ be the number of inequivalent indecomposable representations of A of degree d where d is a positive integer. Then if A has indecomposable representations of arbitrary high degrees and $g_A(d) = \infty$ for an infinite number of integers d, A is said to be of strongly unbounded representation type.

In his paper [1], James P. Jans proved four sufficient conditions²⁵ for an algebra to be of strongly unbounded representation type and, in our previous paper [3], we added two conditions to them but now in this paper we shall prove another sufficient condition for an algebra to be of strongly unbounded representation type:

(7) The graph $G(A_0)$ associated with a two sided ideal $A_0 \subset N$ is $\begin{cases}
P_{j_4}, P_{k_5} \& P_{j_4}, P_{k_5}, P_{k_5} \& P_{j_3}, P_{j_3}, P_{k_4} \& P_{j_3}, P_{k_4}, P_{k_4} \& P_{j_2}, P_{k_2} \& P_{j_2}, P_{k_2} \& P_{j_2}, P_{k_3} \& P_{j_2}, P_{k_3} \& P_{j_2}, P_{k_4} \& P_{k_5}, P_{k_5} \& P_{k_5} \& P_{k_5}, P_{k_5} \& P_{k$

 $P_{k_2}, P_{k_2} \& P_{j_1}, P_{j_1}, P_{k_1} \& P_{j_1}, P_{k_1} \\$

2. First of all we assume that $N^2=0^{4}$ and A is a basic algebra. In order to prove that this condition is sufficient for an algebra to be of strongly unbounded representation type, by the same way as [3] we construct the matrix function R_{cs} , where $c \in k$ and s is an integer,

$$R_{cs}(a) = \begin{bmatrix} X_T(a) & 0 \\ Y(a) & X_B(a) \end{bmatrix}$$
,

as follows:

Let $X_{T}(a)$ be the direct sum of $I_{2s}^{*}X_{j_{4}}(a)$, $I_{6s}^{*}X_{j_{3}}(a)$, $I_{11s}^{*}X_{j_{2}}(a)$ and $I_{5s}^{*}X_{j_{1}}(a)$ and let $X_{B}(a)$ be the direct sum of $I_{4s}^{*}X_{k_{5}}(a)$, $I_{9s}^{*}X_{k_{4}}(a)$, $I_{5s}^{*}X_{k_{3}}(a)$, $I_{8s}^{*}X_{k_{2}}(a)$ and $I_{2s}^{*}X_{k_{1}}(a)$ where $X_{j_{p}}(a)$ and $X_{k_{q}}(a)$ are obtained by the same way as [1] or [3].

Next we put

¹⁾ T. Yoshii [3].

²⁾ James P. Jans [1] or T. Yoshii [3].

³⁾ From now on we use same notations as [1] or [3].

⁴⁾ James P. Jans [1] for the case where $N^2 \neq 0$.