28. *Note on Idempotent Semigroups. IV. Identities of Three Variables*[1)]

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§ 1. *Introduction.* In this short note we shall present the classification of all identities of three variables on idempotent semigroups.

The motivation of taking three for the number of variables has come from the fact that many important identities on idempotent semigroups are written by three or fewer independent variables.

Here only the main two theorems and necessary definitions are given, and the proofs are all omitted. We will study them in detail elsewhere.[2)]

§ 2. *The classification theorem.* Let $X=\{x, y, z\}$. Let F be the free semigroup generated by X. Then an identity is a pair of element of F, say (P, Q), for which we use the notation $P=Q$.

We shall say that an identity $P=Q$ is equivalent to an identity $P'=Q'$, if the following conditions are satisfied:

(1) If S is an idempotent semigroup such that for every homomorphism $h: F\to S$ always $h(P)=h(Q)$, then also $h(P')=h(Q')$.

(2) If S is an idempotent semigroup such that for every homomorphism $h: F\to S$ always $h(P')=h(Q')$, then also $h(P)=h(Q)$.

Theorem 1. *Any identity of three variables on idempotent semigroups is equivalent to one of the following* 18 *distinct properties or identities.*

(1)	*triviality,*	$x=y$;
(2)	*left singularity,*	$xy=x$;
(3)	*right singularity,*	$xy=y$;
(4)	*rectangularity,*	$xyx=x$;
(5)	*commutativity,*	$xy=yx$;
(6)	*left regularity,*	$xyx=xy$;
(7)	*right regularity,*	$xyx=yx$;
(8)	*universality,*	$x=x$;
(9)	*left normality,*	$xyz=xzy$;
(10)	*right normality,*	$xyz=yxz$;
(11)	*normality,*	$xyzx=xzyx$;
(12)	*regularity,*	$xyzx=xyxzx$;
(13)	*left quasi-normality,*	$xyz=xzyz$;

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2) This is an abstract of the paper which will appear elsewhere.