138. On a Generalization of the Concept of Functions. II

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In our previous paper [1], we have introduced the notion of *hyperfunctions* on C^ω-manifolds by means of boundary values of analytic functions as a generalization of the concept of functions, and sketched the theory thereof in case of dimension 1 (the theory of *hyperfunctions of a single variable*). The purpose of the present and subsequent papers is to give the outline of the theory in case of dimensions >1 (the theory of *hyperfunctions of several variables*).[1]

§1. *Distributions of a sheaf.* Let X be a topological space. We denote with $\mathfrak{L}(X)$ the totality of open sets of X. Let $\mathfrak{F}$ be a sheaf of modules over X. For any $D\in\mathfrak{L}(X)$ and $n=0,1,2,\cdots$, we denote as usual the n-cohomology group of D with coefficients in $\mathfrak{F}$ with $H^n(D,\mathfrak{F})$. $H^0(D,\mathfrak{F})$ is the section-module of $\mathfrak{F}$ over D.

Let S be a closed subset of X. For any $D\in\mathfrak{L}(X)$ and $n=0,1,2,\cdots$, we define $G^n(S,D,\mathfrak{F})$ as follows: $G^0(S,D,\mathfrak{F})$ and $G^1(S,D,\mathfrak{F})$ are to mean the kernel and cokernel of the natural homomorphism $H^0(D,\mathfrak{F})\to H^0(D-S,\mathfrak{F})$ respectively, and for $n\geq 2$ we put $G^n(S,D,\mathfrak{F})=H^{n-1}(D-S,\mathfrak{F})$.

For $D\supset D'(D',D\in\mathfrak{L}(X))$ we have the natural homomorphism $\rho^n_{D'D}$: $G^n(S,D,\mathfrak{F})\to G^n(S,D',\mathfrak{F})$. For each n, $(\{G^n(S,D,\mathfrak{F})\}_{D\in\mathfrak{L}(X)}, \{\rho^n_{D'D}\}_{D',D\in\mathfrak{L}(X)})$ constitutes a pre-sheaf over X. We shall denote with $\mathrm{Dist}^n(S,X,\mathfrak{F})$ the sheaf over X determined by this pre-sheaf. $\mathrm{Dist}^n(S,X,\mathfrak{F})$ has the stalk 0 at any point on $X-S$, and if $X'\in\mathfrak{L}(X)$, $X'\supset S$, the natural homomorphism $\mathrm{Dist}^n(S,X,\mathfrak{F})\to\mathrm{Dist}^n(S,X',\mathfrak{F})$ is clearly bijective. In identifying these $\mathrm{Dist}^n(S,X',\mathfrak{F})$, we shall denote the sheaf over S thus determined by $\mathrm{Dist}^n(S,\mathfrak{F})$.

Definition 1. We call each element of $H^0(S,\mathrm{Dist}^n(S,\mathfrak{F}))=H^0(X,\mathrm{Dist}^n(S,X,\mathfrak{F}))$ an *$\mathfrak{F}$-distribution of degree n* over S.

It is clear that we have the natural homomorphism:

$$(1)\qquad G^n(S,X,\mathfrak{F})\to H^0(S,\mathrm{Dist}^n(S,\mathfrak{F}))$$

which is bijective for $n=0$.

Example 1. For $S=X$, we have $\mathrm{Dist}^0(S,\mathfrak{F})=\mathfrak{F}$, $H^0(S,\mathrm{Dist}^0(S,\mathfrak{F}))=H^0(X,\mathfrak{F})$, while $\mathrm{Dist}^n(S,\mathfrak{F})=0$ for $n\geq 1$.

Now let $\{X',\mathfrak{F}',S'\}$ be another triple consisting of a topological space X', a sheaf of modules $\mathfrak{F}'$ over X', and a closed set S' of X'.

1) We have explained our theory, including the case of several variables, in [2] in Japanese. An English account will soon appear in J. Fac. Sci. Univ. Tokyo.