136. On the Singular Integrals. III*)

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1. Let $f(x)$ be a real or complex valued measurable function over $(-\infty, \infty)$, and $\tilde{f}(x)$ the Hilbert transform of $f(x)$, that is

$$\tilde{f}(x)=\lim_{\eta\to 0}\int_{|x-t|>\eta}\frac{f(t)}{x-t}dt. \tag{1.1}$$

The property of this singular integral has been studied by many authors. In particular we feel interest in the result of L. H. Loomis [5] and H. Kober [3]. Other references will be found in E. C. Titchmarsh [6].

We introduce instead of the ordinary Lebesgue measure the following one

$$\mu(\alpha, x)=\int_0^x\frac{dt}{1+|t|^\alpha}, \quad (\alpha\geqq 0) \tag{1.2}$$

and we consider the Hilbert transform for the function of the class $L^p_\mu(p\geqq 1)$ which is the set of $f(x)$ to be measurable with respect to the $d\mu$ and such as

$$\int_{-\infty}^{\infty}|f(x)|^p d\mu(x)=\int_{-\infty}^{\infty}\frac{|f(x)|^p}{1+|x|^\alpha}dx<\infty. \tag{1.3}$$

Clearly the measure function (1.2) plays a role of convergence factor and this enables us to treat the more extensive class of functions. The purpose of this paper is the systematic treatment of the Hilbert operator from a point of view of linear operation. We may use the same notation of constant at each occurrence.

2. The theorems on interpolation of the operation which we need are three. Definitions which we do not state here will be found in the paper of A. Zygmund [7]. Let R and S be two spaces—for simplicity Euclidean spaces—with non-negative and completely additive measure μ and ν respectively. In a previous paper the author [4, I] has extended the definition of A. Zygmund to the case where $\mu(R)$ and $\nu(S)$ are infinite. We need two more theorems which are due to A. P. Calderón and A. Zygmund [1]. These concern with the ordinary Lebesgue measure and we can state in the following form:

Theorem. *Suppose that $\mu(R)$ and $\nu(S)$ are both infinite and that a quasi-linear operation $\tilde{f}=Tf$ is simultaneously of weak type* (1,1) *and* (p, p) $(p>1)$. *Then $\tilde{f}=Tf$ is defined for every f such that*

*) Here we state the result without proof. The detailed argument will appear in the Jour. Faculty Sci. Hokkaidô Univ.