70. *A Method of Solution of Linear Partial Differential Equations of the Second Order with Function Coefficients*

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Although the linear partial differential equations of the second order with constant coefficients of the terms of the second order have already been solved [1, 6], it seems to have not yet done to solve the linear partial differential equations of the second order with function coefficients of the terms of the second order. In this note, a method will be proposed for reducing the latter problem to the former succeeding in solving the linear partial differential equations of the second order with function coefficients:

$$g^{ij}(x)\frac{\partial^2 u}{\partial x^i \partial x^j}+b^l(x)\frac{\partial u}{\partial x^l}+c(x)u=f(x) \tag{0.1}$$

under a certain condition for the function coefficients. The basic principle is to transform the parallel coordinates (x^l) to the II-*geodesic parallel coordinates* (ξ^l) of Prof. T. Takasu [2-4] and to render (0.1) to the form:

$$\varepsilon_l \frac{\partial^2 u}{\partial \xi^l \partial \xi^l}+\bar{b}^l(\xi)\frac{\partial u}{\partial \xi^l}+\bar{c}(\xi)u=\bar{f}(\xi),\ (\varepsilon_l=\pm 1). \tag{0.2}$$

I. Recapitulation of extended affine geometry of Prof. T. Takasu

1. II-Geodesic curves in an affine space A^n. Let (x^l), $(l=1, 2, \cdots, n)$ be parallel coordinates of a point in an *affine space* A^n. Set

$$\omega^l \overset{\text{def}}{=} \omega^l_m(x)dx^m, \quad (l, m, \cdots, p, \cdots=1, 2, \cdots, n), \tag{1.1}$$

where the Pfaffians are assumed to be *not exact* (*an-holonomic*) in general. Throughout this paper, we assume that the affine space A^n is a differentiable manifold of class C^ν(ν=positive integer or $\nu=\infty$ or $\nu=\omega$) and that ω^l are linearly independent:

$$|\omega^l_m(x)| \neq 0 \text{ in } A^n. \tag{1.2}$$

The assumption (1.2) *will be justified by the actual existence of extended affine transformation* in Art. 2.

N.B. We can apply our theory also to the case of the atlas of differentiable manifolds in the sense of S. S. Chern [7] and C. Ehresmann [8].

For the given $\omega^l_m(x)$, we introduce $\Omega^q_l(x)$ by the conditions

$$\Omega^p_l \omega^l_q=\delta^p_q, \quad \Omega^l_q \omega^p_l=\delta^p_q, \tag{1.3}$$

and set