46. On Stable Functional Cohomology Operations

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As is well known, the functional primary cohomology operations[1] are inevitably related to the secondary cohomology operations.[2] Recently J. F. Adams[3] has given an axiomatic characterization for stable secondary operations with its important applications. It seems, then, natural and useful indeed to give a similar axiomatic formulation for stable functional operations, and it is our objective.

We follow Adams' notations.[4] Let p be a prime; let A be the Steenrod algebra[5] over Z_p. An A-module is to be a graded left module over the graded algebra A. Let us write $H^*(X)$ for $\sum_q H^q(X, Z_p)$ and $H^+(X)$ for $\sum_{q>0} H^q(X, Z_p)$; then they are A-modules.

Let C_0, C_1 be free A-modules of locally finite type such that $(C_i)_q=0$ if $q<i$ $(i=0, 1)$. Let (d, v) be a pair of an A-map $d: C_1\to C_0$ of degree zero and a homogeneous element v of C_1. We call φ a stable functional primary cohomology operation associated with (d, v), if it satisfies the following axioms.

AXIOM 1. $\varphi(f, \varepsilon)$ is defined for each pair of a map $f: Y\to X$ and an A-map $\varepsilon: C_0\to H^+(X)$ of degree $m\geqq 1$ such that $f^*\varepsilon=0$ and $\varepsilon d=0$.

Such a map ε is determined by its values on the elements of an A-base of C_0. It therefore corresponds to a set of elements of $H^+(X)$. In particular, if C_0, C_1 are free on one given generator e_i $(i=0, 1)$ respectively and $de_1=ae_0$ $(a\in A)$, then we write $u=\varepsilon e_0$ and $\varepsilon d=0$ means $au=0$; we may thus consider the operation φ associated with (d, e_1) as a function of one variable u for a fixed map f, where u runs over a subset of $H^+(X)$. In this case we write $a_f(u)$ for $\varphi(f, \varepsilon)$ as usual.

For the next axiom, set $\deg(v)=\nu$, let $\lambda: C_0\to H^+(Y)$ run over the A-maps of degree $m-1$, and let $L^{m+\nu-1}(d, v; f)$ be the set of elements of the form $\lambda dv+f^*x$ $(x\in H^{m+\nu-1}(X))$.

1) N. E. Steenrod: Cohomology invariants of mappings, Ann. Math., **50**, 954-988 (1949); F. P. Peterson: Functional cohomology operations, Trans. A. M. S., **86**, 197-211 (1957).

2) J. Adem: The iteration of the Steenrod squares in algebraic topology, Proc. Nat. Acad. Sci. U. S. A., **38**, 720-726 (1952); F. P. Peterson and N. Stein: Secondary cohomology operations: two formulas, Amer. J. M., **81**, 281-305 (1959).

3) J. F. Adams: On the nonexistence of elements of Hopf invariant one, Bull. A. M. S., **64**, 279-282 (1958).

4) Loc. cit., 3).

5) H. Cartan: Sur l'itération des opérations de Steenrod, Comment. Math. Helv., **29**, 40-58 (1955).