31. *Convergence to a Stationary State of the Solution of Some Kind of Differential Equations in a Banach Space*

By Hiroki TANABE

(Comm. by K. KUNUGI, M.J.A., March 13, 1961)

1. Introduction. The purpose of this note is to investigate the behaviour at $t=\infty$ of the solution $x(t)$ of some type of differential equation

$$dx(t)/dt=A(t)x(t)+f(t), \tag{1.1}$$

in a Banach space $\mathfrak{X}$. Roughly speaking, if $A(t)$ and $f(t)$ have some properties and if both of them converge in some sense as $t\rightarrow\infty$, then the solution $x(t)$ also converges to some element as $t\rightarrow\infty$.

2. Assumptions and the theorem. By Σ we denote the set of all the complex numbers λ satisfying $-\theta\leqq\arg\lambda\leqq\theta$, where θ is a fixed angle with $\pi/2<\theta<\pi$.

Assumption 1°. For each t, $0\leqq t<\infty$, $A(t)$ is a closed additive operator which maps a dense subset of $\mathfrak{X}$ into $\mathfrak{X}$. The resolvent set $\rho(A(t))$ of $A(t)$, $0\leqq t<\infty$, contains Σ and the inequality

$$||(\lambda I-A(t))^{-1}||\leqq M/(|\lambda|+1) \tag{2.1}$$

is satisfied for each $\lambda\in\Sigma$ and $t\in[0,\infty)$, where M is a positive constant independent of λ and t.

2°. The domain D of $A(t)$ is independent of t and the bounded operator $A(t)A(s)^{-1}$ is Hölder continuous in t in the uniform operator topology for each fixed s;

$$||A(t)A(s)^{-1}-A(r)A(s)^{-1}||\leqq K|t-r|^{\rho},$$
$$K>0,\ 0<\rho\leqq 1,\ 0\leqq t,\ r<\infty, \tag{2.2}$$

where K and ρ are positive constants independent of t, r and s.

3°. $f(t)$ is uniformly Hölder continuous in $0\leqq t<\infty$:

$$||f(t)-f(s)||\leqq F(t-s)^{\gamma},\ F>0,\ 0<\gamma\leqq 1,\ 0\leqq s,\ t<\infty, \tag{2.3}$$

where F and γ are some constants independent of s and t.

4°. There exist a closed operator $A(\infty)$ with domain D and an element $f(\infty)$ of $\mathfrak{X}$ such that

$$||(A(t)-A(\infty))A(0)^{-1}||\rightarrow 0,\quad ||f(t)-f(\infty)||\rightarrow 0 \tag{2.4}$$

as $t\rightarrow\infty$.

Theorem. Under the assumptions made above, the solution $x(t)$ of (1.1) converges to some element as $t\rightarrow\infty$. The limit $x(\infty)$ belongs to D and satisfies

$$A(\infty)x(\infty)+f(\infty)=0. \tag{2.5}$$

Moreover, $dx(t)/dt$ tends to 0 as $t\rightarrow\infty$.

It might be possible to make a similar observation about the kind of equations investigated by Prof. T. Kato. Such equations