54. On the Definition of the Knot Matrix

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The knot matrix defined in [3] for alternating knots can be defined for any knots or links. To do this, we introduce three indices η, d, ε for each crossing point.[1]

Let a regular projection K in S^2 of a knot k have m of the second kind of the loops which divides S^2 into $m+1$ domains, $E_1, E_2, \cdots, E_{m+1}$. Let us denote $(E_i \smile \dot{E}_i) \frown K = K_i$. Then the regions contained in E_i can be classified into two classes, black and white (cf. Lemma 1.10 [3]). For the sake of brevity, any crossing point in K_i is denoted by $c_{i,\lambda}$, any white region in E_i is denoted by $X_{i,\mu}$ and w_i denotes the number of the white regions in E_i.

[Definition 1] For any $c_{i,\lambda}$, the first index η_i is defined as $+1$ or -1 as is shown in the following figure.

$\eta_i = +1$ (a) $\qquad$ $\eta_i = -1$ (b)

(It should be noted that the orientation of k is irrelevant to the definition.)

As usual, two corners among four corners meeting at a crossing point are marked with dots [3].

[Definition 2] For any $c_{j,\lambda}$, the second index d is defined as follows.

(1) $d_{X_{i,\mu}}(c_{j,\lambda}) = 1$ or 0 according as the $c_{j,\lambda}$-corner of $X_{i,\mu}$ is dotted or undotted.

(2) $d_{X_{i,\mu}}(c_{j,\lambda}) = 0$ if $c_{j,\lambda}$ does not lie on $\dot{X}_{i,\mu}$.

If $c_{j,\lambda}$ lies on $\dot{X}_{j,\mu}$, then the third index $\varepsilon_{X_{j,\mu}}(c_{j,\lambda})$ is defined as $+1$ or -1 according as the $c_{j,\lambda}$-corner of $X_{j,\mu}$ is dotted or undotted.

By means of these indices, the knot matrix of K can be defined.

[Definition 3] The knot matrix $M = (M_{ij})_{i,j=1,2,\cdots,m+1}$ is defined as follows:

$$
(1) \qquad
\begin{aligned}
M_{ii} &= (a_{pq}^{(i)})_{p,q=1,\cdots,w_i} \\
-a_{pq}^{(i)} &= \sum_{c_{i,\lambda} \in \dot{X}_{i,p} \frown \dot{X}_{i,q}} \eta_i(c_{i,\lambda})\, d_{X_{i,p}}(c_{i,\lambda}), \quad (p \neq q), \\
a_{pp}^{(i)} &= -\sum_{\substack{p=1 \\ p \neq q}}^{w_i} a_{pq}.
\end{aligned}
$$

1) For symbols and notations, see [3].