46. *Finite-to-one Closed Mappings and Dimension. IV*

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We have published in the Proceedings three notes under this title with sketches of proofs or no proofs. The detail of the content stated there will be published, under the title 'Mappings of finite order and dimension theory', in forthcoming Japanese Journal of Mathematics. We have mainly been concerned with finite-to-one closed mappings between metric spaces. We shall notice in this note a deep relation between finite-to-one closed mappings defined on non-metrizable spaces and the inductive dimension. The detail of the content of the present note will be published in another place. In the following n denotes a non-negative integer.

Let R be a topological space. We define inductively the small and the large inductive dimension of R, $\operatorname{ind} R$ and $\operatorname{Ind} R$, as follows. For the empty set ϕ let $\operatorname{ind}\phi=\operatorname{Ind}\phi=-1$. We call $\operatorname{ind} R\leq n$, if for any point x and an open set G with $x\in G$ there exists an open set H with $x\in H\subset G$ such that $\operatorname{ind}(\overline{H}-H)\leq n-1$. We call $\operatorname{Ind} R\leq n$, if for any pair $F\subset G$ of a closed set F and an open set G there exists an open set H with $F\subset H\subset G$ such that $\operatorname{Ind}(\overline{H}-H)\leq n-1$.

Let $\mathfrak{F}=\{F_\alpha;\ \alpha\in A\}$ be a collection of subsets of R. Then the order of $\mathfrak{F}$ at x, $\operatorname{order}(x,\mathfrak{F})$, is the number of elements of $\mathfrak{F}$ which contain x. The order of $\mathfrak{F}$, $\operatorname{order}\mathfrak{F}$, is the supremum of $\{\operatorname{order}(x,\mathfrak{F});\ x\in R\}$. The star of H, a subset of R, with respect to $\mathfrak{F}$, $\mathrm{S}(H,\mathfrak{F})$, is the sum of $F\in\mathfrak{F}$ with $H\frown F\neq\phi$. Let S be a subset of R. Then the restriction of $\mathfrak{F}$ to S, $\mathfrak{F}\wedge S$, is the collection $\{F_\alpha\frown S;\ \alpha\in A\}$. Let $\mathcal{F}=\{\mathfrak{F}_\lambda;\ \lambda\in\Lambda\}$ be a system of collections of subsets of R. Then the order of $\mathcal{F}$, $\operatorname{order}\mathcal{F}$, is the supremum of $\{\operatorname{order}\mathfrak{F}_\lambda;\ \lambda\in\Lambda\}$.

Definition 1. Let $\mathcal{F}=\{\mathfrak{F}_\lambda;\ \lambda\in\Lambda\}$ be a system of coverings of a topological space R. $\mathcal{F}$ is called to *follow out the topology of R locally, globally* and *fully*, if the following conditions are respectively satisfied.

(1) For any point x of R and any open set G with $x\in G$ there exists $\lambda\in\Lambda$ with $\mathrm{S}(x,\mathfrak{F}_\lambda)\subset G$.

(2) For any pair $F\subset G$ of a closed set F and an open set G of R there exists $\lambda\in\Lambda$ with $\mathrm{S}(F,\mathfrak{F}_\lambda)\subset G$.

(3) For any open covering $\mathfrak{G}$ of R there exists $\lambda\in\Lambda$ such that $\mathfrak{F}_\lambda$ refines $\mathfrak{G}$.

Definition 2. Let $\mathcal{F}=\{\mathfrak{F}_\lambda=\{F_\alpha;\ \alpha\in A_\lambda\};\ \lambda\in\Lambda\}$ be a system of coverings of a topological space R. $\mathcal{F}$ is called a directed family (with