133. Some Analytical Properties of the Spectra of Normal Operators in Hilbert Spaces

By Sakuji INOUE

Faculty of Education, Kumamoto University

(Comm. by K. KUNUGI, M.J.A., Nov. 13, 1961)

Definition. Let $\mathfrak{H}$ be the complex abstract Hilbert space which is complete, separable and infinite dimensional; and let N_j, $j=1,2,\cdots,p$, be bounded normal operators in $\mathfrak{H}$. We then define a complex-valued function $S(f,g;\lambda)$ of a complex variable λ by

$$S(f,g;\lambda)=\left(\left(\sum_{j=1}^{p}\sum_{\alpha=1}^{m_j}c_{\alpha j}(\lambda I-N_j)^{-\alpha}\right)f,g\right),\quad\left(f\in\bigcap_{j=1}^{p}\mathfrak{D}((\lambda I-N_j)^{-m_j}),\ g\in\mathfrak{H}\right),$$

under the assumption that the set of accumulation points of the point spectrum of each N_j is an at most denumerably infinite set.

Though f here is arbitrarily chosen in the domain $\bigcap_{j=1}^{p}\mathfrak{D}((\lambda I-N_j)^{-m_j})$ so that the function S is significant, the domains of f in the results of integrations of S along such curves as will afterwards be defined are extended respectively: because the respective integrals of $\sum_{j=1}^{p}\sum_{\alpha=1}^{m_j}c_{\alpha j}(\lambda I-N_j)^{-\alpha}$ are reduced to simplified operators as we shall see later on.

As will afterwards be verified immediately from the integral expressions and the expansions of N_j, $j=1,2,\cdots,p$, the following statements are valid:

1° $S(f,g;\lambda)$ is regular in the intersection of all resolvent sets of N_j, $j=1,2,\cdots,p$, only;

2° every isolated eigenvalue of N_j, $j=1,2,\cdots,p$, is a pole with order m_j of $S(f,g;\lambda)$;

3° though every accumulation point of isolated eigenvalues of each N_j is a non-isolated essential singularity of $S(f,g;\lambda)$ in the sense of the function theory, $S(f,g;\lambda)$ has the principal part of the expansion at that point when and only when it belongs to the point spectrum of the N_j;

4° every point belonging to the union of the continuous spectra of N_j, $j=1,2,\cdots,p$, is a non-regular point of $S(f,g;\lambda)$, but not a usual singularity in the sense of the function theory unless it is an accumulation point of isolated eigenvalues of any one of N_j, $j=1,2,\cdots,p$.

In particular, we are interested in the case where $S(f,g;\lambda)$ has a denumerably infinite set of non-isolated essential singularities. We shall discuss the integral of $S(f,g;\lambda)$ along a rectifiable closed Jordan curve comprising those denumerably infinite essential singularities