28. On the Propagation of Regularities of Solutions of Partial Differential Equations

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§1. Introduction and Theorem. Let $P\left(x, \frac{\partial}{\partial x}\right)$ be a partial differential operator of order m with C^∞-coefficients defined in a spherical neighbourhood $S(a, h)$ of a point a with radius h in R^n. Furthermore let Ω be a domain in R^n containing the point a as a point on its boundary.

Then we define the (a, Ω)-regularities as follows:

Definition 1. We say that $P\left(x, \frac{\partial}{\partial x}\right)$ is (a, Ω)-*regular* if for any r and any integer s, there are positive numbers l, t such that every distribution solution u of the equation $P(x, D)u=0$ defined in $S(a, r)$ is in $C^s(S(0, l))$, whenever $u \in C^t(S(a, r) \cap (R^n - \bar{\Omega}))$, where t depends upon r, l and s, but l depends only upon r and the derivatives of the coefficients of $P(x, D)$.

Definition 2. We say that $P\left(x, \frac{\partial}{\partial x}\right)$ is *hypo*-(a, Ω)-*regular* if for any r, there is a positive number l such that every distribution solution u of the equation $P(x, D)u=0$ defined in $S(a, r)$ is in $C^\infty(S(a, l))$, whenever $u \in C^\infty(S(a, r) \cap (R^n - \bar{\Omega}))$.

In the previous paper [4], I considered the case where $P(x, D)$ has constant coefficients and $\Omega = \{x \mid (x, \xi) > 0\}$ and showed that let $n \geqq 3$ then for a homogeneous polynomial p and for any q with order $\leq m-1$, $P=p+q$ is $(0, \Omega)$-regular if and only if the roots z of $p(z\xi + \eta) = 0$ $(\xi \perp \eta)$ can't cut the real axis and the real roots are simple.

On the other hand in variable coefficients L. Hörmander considered every interesting theorems some of which showed the sufficient condition for P to be the hypo-Ω-regular [1].

In the present note under the same conditions as Hörmander's we shall generalize the distinctive feature of regularities mentioned above to variable coefficients as follows: denoting $p^{(j)}(x, \xi) = \frac{\partial}{\partial \xi_j} p(x, \xi)$, $\left(p_j(x, \xi) = \frac{\partial}{\partial x_j} p(x, \xi)\right.$ and $\bar{p}(x, \xi) = \overline{p(x, \xi)}$, we have

Theorem. Let the principal part $p(x, D)$ of $P(x, D)$ have the property such that for some real valued function $\phi \in C^\infty(S(0, h))$ with