23. *Inversive Semigroups. I*

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§1. **Introduction.** A semigroup S is called *inversive* if it satisfies the following

$$(\mathrm{C})\left\{\begin{array}{l}\text{(1) } S \text{ has an idempotent, and the totality } I \text{ of idempotents} \\ \quad \text{of } S \text{ is a subband of } S. \\ \text{(2) For any element } x \text{ of } S, \text{ there exists an element } x^* \\ \quad \text{such that } xx^*=x^*x \text{ and } xx^*x=x. \end{array}\right.$$

In this case, for any element x of S there exists one and only one element y such that $xy=yx$, $xyx=x$ and $yxy=y$. Such a y is called the *relative inverse* of x, and denoted by x^{-1}. Now, let $S(e)=\{x: xx^{-1}=e\}$ for each element e of I. Then, it is easy to see that each $S(e)$ is a subgroup of S and S is the class sum of all $S(e)$ (A. H. Clifford [1] has shown that a semigroup satisfying the condition (1) of (C), which is called a *semigroup admitting relative inverses*, is the class sum of subgroups). Therefore, inversive semigroups are not too far away from groups. Actually, as a special case, the author has proved in [4] that if I is a rectangular band then all $S(e)$ are isomorphic to each other and S is isomorphic to the direct product of an $S(e)$ and I. An inversive semigroup in which the totality of idempotents is a rectangular subband is called an *(R)-inversive semigroup.*

Now, it is clear that any (R)-inversive semigroup satisfies the following

(C. 1) If $xx^{-1}=e$ and if f is an idempotent such that $f\leqq e$ (i.e. $fe=ef=f$), then $fx=xf$.

However, an inversive semigroup satisfying the condition (C. 1) is not necessarily (R)-inversive. By a *strictly inversive semigroup,* we shall mean an inversive semigroup satisfying the condition (C. 1). The main purpose of this paper is to present a structure theorem for strictly inversive semigroups, and some relevant matters. The proofs are omitted and will be given in detail elsewhere.[1)]

§2. **The structure of strictly inversive semigroups.** Let G be a semigroup. If there exist a band Ω and a collection $\{G_\alpha : \alpha\in\Omega\}$ of subsemigroups of type $\mathfrak{T}$ such that

(i) $G=\bigcup\{\alpha_\alpha : \alpha\in\Omega\}$,

(ii) $G_\beta\cap G_\gamma=\phi$ for $\beta\neq\gamma$

and (iii) $G_\beta G_\gamma\subset G_{\beta\gamma}$.

1) This is an abstract of the paper which will appear elsewhere.