16. *Time Change and Killing for Multi-Dimensional Reflecting Diffusion*

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1. Introduction. H. Tanaka and the author have defined in [8] *the local time on the boundary* for multi-dimensional reflecting diffusion. We show that this local time serves as a time change function in reducing the diffusion to *the Markov process on the boundary* introduced by T. Ueno [9]. This fact has been conjectured by him in [9]. In order to treat more general cases with killing (mass defect), we prove some general results on Markov processes. We also construct the diffusion with killing and sojourn on the boundary, which is an extension of the results obtained by K. Ito and H. P. McKean, Jr. [5] in one dimension and by N. Ikeda [3] in two dimensions.

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2. Definitions and notations. We use the definitions and notations of E. B. Dynkin [1] concerning Markov processes, unless specifically mentioned. Suppose a (temporally homogeneous) Markov process $X=(x_t, \zeta, \mathcal{M}_t, \mathbf{P}_x, \theta_t)$ with state space $(E, \mathcal{B})$. We consider the following conditions:

M 1. E is a locally compact Hausdorff space with a countable base and $\mathcal{B}$ is the σ-algebra generated by the open sets.

M 2. $\mathbf{P}_x(\zeta>0)=1$ for all $x\in E$.

M 3. X is right-continuous.

M 4. X has the strict Markov property.

M 5. If $\tau_n(\omega)\uparrow\tau(\omega)<\zeta(\omega)$ for all $\omega\in B$, where τ_n are random variables independent of the future, then, for all x, $x_{\tau_n}\to x_\tau$ (a.e. $B, \mathbf{P}_x$).

M 6. $\mathcal{M}_{t+0}=\mathcal{M}_t$.

M 7. $\overline{\mathcal{M}}_t=\mathcal{M}_t$.[1)]

If X satisfies the above conditions, X is a standard process in the sense of Dynkin. We call $\varphi_t(\omega)$ $(\omega\in\Omega_t)$ a *continuous* [*right-continuous*] *non-negative additive functional of* X, if it satisfies the following five conditions:

A 1. $\varphi_s(\omega)+\theta_s\varphi_t(\omega)=\varphi_{s+t}(\omega)$ for all $\omega\in\Omega_{s+t}$;

A 2. φ_t is $\mathcal{R}_t$-measurable;[2)]

1) $\overline{\mathcal{M}}_t$ is the family of B such that, for every finite measure μ, there exist B_1 and $B_2\in\mathcal{M}_t$ satisfying $B_1\subseteqq B\subseteqq B_2$ and $\mathbf{P}_\mu(B_1)=\mathbf{P}_\mu(B_2)$.

2) We put $\overline{\mathcal{R}}_t=\mathcal{N}_{t+0}\cap\mathcal{N}^*$.