36. On the Absolute Nörlund Summability Factors of a Fourier Series

By H. P. DIKSHIT

(Comm. by Kinjirô KUNUGI, M.J.A., March 12, 1963)

1.1. *Definitions.* Let $\sum u_n$ be a given infinite series with the sequence of partial sums $\{s_n\}$. Let $\{p_n\}$ be a sequence of constants, real or complex, and let us write

$$P_n = p_0 + p_1 + \cdots + p_n; \quad P_{-1} = p_{-1} = 0.$$

The sequence-to-sequence transformation:

$$t_n = \frac{1}{P_n}\sum_{\nu=0}^{n} p_{n-\nu}s_\nu = \frac{1}{P_n}\sum_{\nu=0}^{n} P_{n-\nu}u_\nu, \quad (P_n \neq 0), \tag{1.1.1.}$$

defines the sequence $\{t_n\}$ of Nörlund means of the sequence $\{s_n\}$, generated by the sequence of coefficients $\{p_n\}$. The series $\sum u_n$ is said to be summable (N, p_n) to the sum s if $\lim_{n\to\infty} t_n$ exists and is equal to s, and is said to be absolutely summable (N, p_n), or $|N, p_n|$, if the sequence $\{t_n\}$ is of bounded variation,[1] that is, the series $\sum |t_n - t_{n-1}|$ is convergent. In the special case in which

$$p_n = 1/(n+1) \tag{1.1.2}$$

the Nörlund mean reduces to the Harmonic mean.

Thus summability $|N, p_n|$, where p_n is defined by (1.1.2) is the same as the absolute Harmonic summability.

1.2. Let $f(t)$ be a periodic function, with period 2π, and integrable in the sense of Lebesgue over $(-\pi, \pi)$. Then the Fourier series of $f(t)$ is

$$\sum (a_n \cos nt + b_n \sin nt) = \sum A_n(t). \tag{1.2.1}$$

We write

$$\phi(t) = \frac{1}{2}\{f(x+t) + f(x-t)\},$$

$\tau = [1/t]$, i.e., the greatest integer contained in $1/t$.

K = an absolute constant, not necessarily the same at each occurrence.

2.1. We establish the following theorem.

Theorem. *If $\phi(t) \in BV(0, \pi)$, and $\{\lambda'_n\}$, where $\lambda'_n = \frac{\lambda_n}{n}$, is monotonic increasing then $\sum_{n=1}^{\infty} nA_n(t)/\lambda_n$ is summable $|N, p_n|$, provided $\{p_n\}$ satisfies the following conditions:*

(i) *$\{p_n\}$ is monotomic diminishing, and P_n is monotonic in-*

---

1) Symbolically, $\{t_n\} \in BV$; similarly by '$f(x) \in BV\ (h, k)$' we shall mean that $f(x)$ is a function of bounded variation over the interval (h, k).