104. *Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. VIII*

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On the assumption that $S(\lambda)$ and $R(\lambda)$ are the functions defined in the statement of Theorem 1 [cf. Proc. Japan Acad., Vol. 38, 263–268 (1962)], in the preceding papers we have discussed, under some conditions, the distribution of ζ-points of those functions in the exterior of a suitably large circle with center at the origin and the relation between the two finite exceptional values of those functions for the exterior of that circle, by using the extended Fourier series expansion of the function $\chi(\lambda)$ defined as the sum of the two principal parts of $S(\lambda)$. In the present paper, however, we shall treat, under some conditions, those problems with respect to the derivatives of $R(\lambda)$ and $S(\lambda)$ from a different point of view, by applying the integral expressions of the derivatives of $\chi(\lambda)$.

Theorem 21. Let $S(\lambda)$ and $\{\lambda_\nu\}$ be the same notations as those defined in the statement of Theorem 1; let $R(\lambda)$ be the ordinary part of $S(\lambda)$; let σ be a given positive constant satisfying the inequality $\sup_\nu|\lambda_\nu|<\sigma<\infty$; let $\{z_n\}$ be a set of mutually distinct ζ-points of $R(\lambda)$ such that

$$\left.\begin{array}{l} R(z_n)=\zeta \\ \sigma<|z_n|\leqq|z_{n+1}| \end{array}\right\}\ (n=1,2,3,\cdots),\quad |z_n|\to\infty\ \ (n\to\infty);$$

let

$$\text{(A)}\qquad \frac{R^{(m)}(0)}{R^{(m-1)}(0)}\neq c,\quad \frac{R^{(m+\mu)}(0)}{R^{(m+\mu-1)}(0)}=c\ \ (\mu=1,2,3,\cdots),$$

where c is a non-zero complex constant and m is a positive integer; let r_n, $n=1,2,3,\cdots$, be positive numbers satisfying the conditions

$$\text{(B)}\qquad \left|\frac{R(z_n+r_ne^{i\theta})-R(z_n)}{r_ne^{i\theta}}-R'(z_n)\right|<\varepsilon,\quad \inf_n\{r_n|z_n|^{m+1-\varepsilon}\}\neq 0$$

for a given positive number ε less than 1; let $\widetilde{R}_\alpha(\lambda)=R(\lambda)-P_\alpha(\lambda)$ and $\widetilde{S}_\alpha(\lambda)=S(\lambda)-P_\alpha(\lambda)$, where

$$P_\alpha(\lambda)=\sum_{j=\alpha}^{m}\frac{1}{j!}\{R^{(j)}(0)-cR^{(j-1)}(0)\}\lambda^j\ \ (\alpha=1,2);$$

and let Γ_n denote the circle $|\lambda-z_n|=r_n$. Then any z_n in the set $\{z_n\}$ is a $\{(\alpha-1)[R'(0)-cR(0)]+c\zeta\}$-point of $\widetilde{R}'_\alpha(\lambda)$ and there exists a suitably large positive integer L such that in the interior of each of the circles Γ_{L+p}, $p=0,1,2,\cdots$, $\widetilde{S}'_\alpha(\lambda)$ has $\{(\alpha-1)[R'(0)-cR(0)]+c\zeta\}$-points the number of which equals that of $\{(\alpha-1)[R'(0)-cR(0)]+c\zeta\}$-